KINEMATICS OF MACHINES.

CHAPTER I.

INTRODUCTORY CONSIDERATIONS.

1. **Study of Machines.** — In general the study of a Machine involves problems of three distinct kinds. We may first of all consider from a geometrical point of view the motion of any part of the machine with reference to any other part, without taking account of any of the forces acting on such parts. Or, the action of the forces impressed on the parts of the machine, and of the forces due to its own inertia or to the weight of its parts, may be dealt with, and the resulting transformations of energy may be determined. A third branch of the theory of machines treats of the action of these loads and forces in producing stresses and strains in the materials employed in the construction of the machine, and discusses the sizes, forms, and proportions of the various parts which are required either to insure proper strength while avoiding waste of material, or to make the machine capable of doing the work for which it is being designed.

The science dealing with the first-named class of problem is termed the *Kinematics of Machines*, which we may define as being that science which treats of the relative motion of the parts of machines, without regard to the forces producing such motions, or to the stresses and strains produced by such forces.

With this limitation, in the case of almost all bodies forming portions of machines, it is possible to neglect any deformation they may undergo in working, and in studying the Kinematics of Machines we may at once apply to machine problems the results obtained by the study of the motion of rigid bodies. Important exceptions will present themselves to the reader's mind; for example, ropes, belts, and springs cannot be considered kinematically as being rigid, and many mechanical contrivances involve the use of liquid or gaseous material. Such cases as these will be considered later.

By the term *Machine* we may understand a combination or arrangement of certain portions of resistant material, the relative motions of which are controlled in such a way that some form of available energy is transmitted from place to place, or is transformed into another desired kind. This definition includes under the head of Machines all contrivances which have for their object the transformation or transmission of energy, or the performance of some particular kind of work, and further implies that a single portion of material is not considered as a machine. The so-called *simple machines* in every case involve the idea of more than one piece of material.

A combination or arrangement of portions of material by means of which forces are transmitted or loads are carried without sensible relative motions of the component parts is called a *Structure*.

The term *Mechanism* is often used as an equivalent for the word Machine. It is, however, preferable to restrict its use somewhat, and to employ the word to denote simply a combination of pieces of material having definite relative motions, one of the pieces being regarded as fixed in space. Such a mechanism often represents kinematically some actual machine which has the same number of parts as the mechanism with the same relative motions. The essential difference is that in the case of a machine such parts have to transmit or transform energy, and are proportioned and formed for this end, while in a mechanism the relative motion of the parts only is considered. We may look upon a mechanism, then, as being the ideal or kinematic form of a machine, and our work will be much simplified in most cases if we consider for kinematic purposes the mechanism instead of the machine. Such a substitution is also of the greatest service in the comparison and classification of machines; we shall find in this way that machines, at first sight quite distinct, are really related, inasmuch as their representative mechanisms consist of the same number of parts having similar relative motions, and only differing because a different piece is considered to be fixed in each case.

2. Constrained Motion.—On further consideration of the nature of a Machine as defined above, it will be noted that each part of the machine must have certain definite motions relatively to any other part, such definite motions being repeated again and again during the working of the machine. Thus the motion of a machine-part must be completely *constrained*, that is, the part must be free to move only in the manner desired to produce the required transformation of energy, and for it other unnecessary motions must be rendered impossible. Constrained motion of a body takes place when every point in the body is made to describe some definite and prescribed path. This constraint is effected in general by so forming and connecting the parts that all forces tending to disturb their constrained motion are balanced by stresses set up in the parts themselves. It is assumed, of course, that the machine remains uninjured by such stresses.

3. Pairs of Elements.—The nature of the connection between the parts of a machine will be best understood by taking a simple case and discussing the way in which some form of constrained relative motion of two bodies may be obtained. Suppose, for example, that a piece of material, which we may call a, has to be capable of a motion of translation along a straight line, with reference to another piece, b, and is to have no other relative motion whatever. This must be accomplished by giving these pieces suitable forms. Such an arrangement as that sketched in Fig. 1

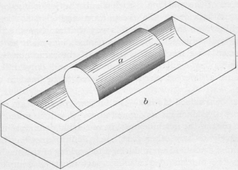

FIG. 1.

would not meet the case, for, although a executes the required movement so long as it remains in the groove formed in b and does not rotate on its axis in the groove, the forms shown do not prevent a leaving the groove in b or rotating in that groove.

It will be found that to attain the desired object some such forms as shown in Fig. 2 must be adopted, and that if this is done, the only possible motion of a relatively to b is that of simple translation along a straight line parallel to the edge of the groove or slot in b. The figure will recall to the reader the appearance of a steam-engine cross-head and its guides, a pair of bodies which have indeed the same relative motion as that described above.

We shall refer to a pair of bodies so formed as to permit of partly or wholly constrained relative motion while in contact as a *pair of elements*, the elements being really the surfaces of contact, or working surfaces, of the pair of bodies. Such pairs are distinguished as being (*a*) *higher pairs* and (*b*) *lower pairs*. Lower pairs may be defined as those in which "the forms of the elements are geometrically identical, the one being solid or full and the other hollow or open" (Reuleaux). This definition involves the idea of surface contact to produce the required partial or complete constraint, while in the case of higher pairs constraint is produced by contact at a sufficient number of lines or

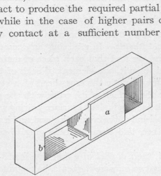

FIG. 2.

points. Mechanically, lower pairing in machinery is preferable, wherever possible. The reason for this is that wear takes place much more rapidly in a case where line or point contact occurs than in the case where surfaces of considerable extent are touching, other conditions being the same.

A pair of elements whose relative motion is completely constrained is said to be *closed*. Thus such a pair as is shown in Fig. 1 is not closed, while that of Fig. 2 is completely closed; for, as has been already pointed out, the only possible relative motion is one of pure translation in a straight line.

The nature of the relative motion of two bodies can only be defined when one of them is considered as being fixed. In the case of a pair of elements *ab*, *a* being fixed while *b* moves, we may have the same relative motion of *a* and *b* as when *b* is fixed while *a* moves, but the pair is said to be inverted, that is, the second element is fixed instead of the first. Examples of such *inversion* of pairs frequently occur in considering actual machines, and it is important to remember that, while inversion of a pair may cause no alteration of the *relative* motion of the elements themselves, it may, and generally does, alter their motion relatively to other bodies.

4. Links and Chains.—In studying any simple mechanism or machine, we find that each piece of material carries, or has formed upon it, one element of each of two or more pairs. Take for example the cross-head of a steam-engine; in addition to the surface which pairs with the guide bar or bars, the block has a cylindrical surface pairing with a similar one on the small end of the connecting-rod, and it thus carries, or links together, two elements belonging to two different pairs.

In general, then, a part of a machine forms a *kinematic link* connecting two or more elements, belonging respectively to two or more pairs, and the whole arrangement or combination of such links is known as a *kinematic chain*. This may or may not have such kinematic properties as to make it available as a mechanism; for we can easily imagine a kinematic chain which does not comply with our definition of a mechanism when one link is fixed. Consider the case of a linkwork formed of five bars, *a b c d e*, jointed at the angles as shown in Fig. 3. Suppose *a* to be fixed, then the motion of *c* or *d* relatively to *a* is not constrained, and such a chain, therefore, is not a mechanism as we have defined it.

It is most important to note, with regard to this point, that the motion of *c* with respect to *b* *is* constrained, i.e., *c* can only have one motion with regard to *b*, that of turning about the axis of the joint connecting them, whereas with respect to a, c can be made to move in any number of different ways, depending in this case on the force or forces applied to the different bars. The motion of c with respect to a is therefore not constrained. This fact is illustrated in Fig. 3, where it is seen that if the links b and e take up the positions b' and e', c and d may be either at c' and d', or at c'' and d''. Such a kinematic chain as this is said not to be *closed*, and we define a *closed chain* as a series of links

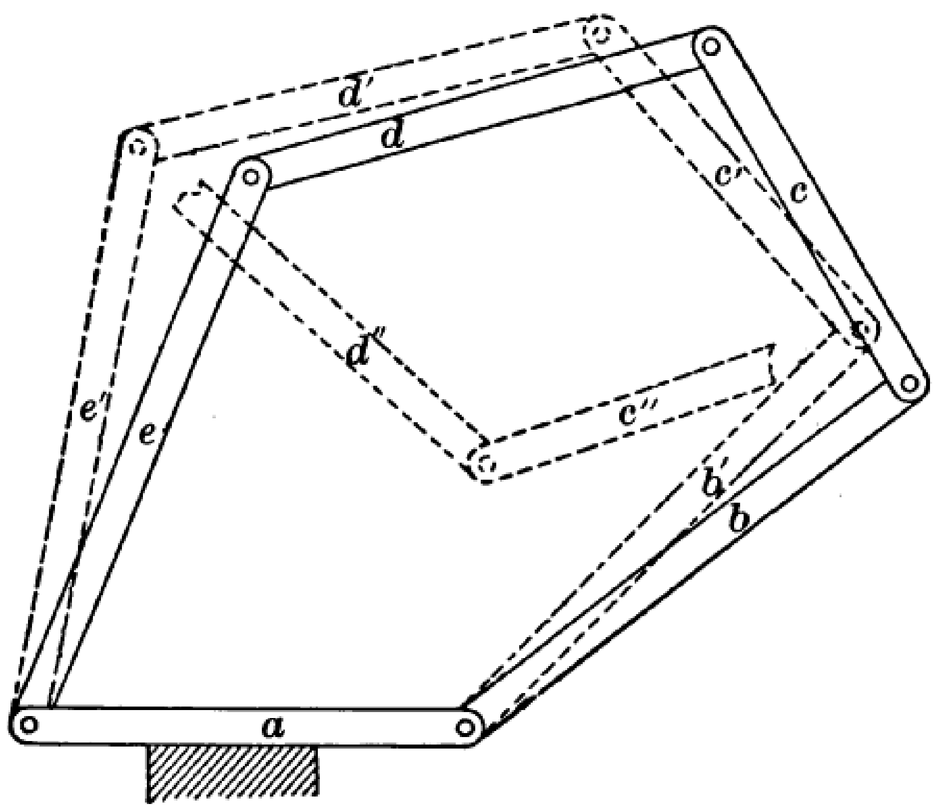

FIG. 3.

so connected that each of them has only one definite motion relatively to any other link. Thus if one link be fixed, the motion of any other can be determined. A closed chain having one link fixed is then equivalent to a mechanism.

The various ways in which closure is obtained in pairs and in chains will be discussed later.

A chain of which each link carries two elements is termed a *simple chain*, for a link cannot have a less number of elements than two. If, however, any link or links have three or more elements respectively belonging to three or more pairs, the chain is said to be *compound*. In some ways compound chains present more difficulties than do simple chains, but the kinematics of both kinds may be studied by exactly the same methods. Fig. 4 shows a closed compound chain, which has been suggested as a straight-line motion. It will be seen that the link *a* is fixed, and that *b* carries one element of each of the pairs

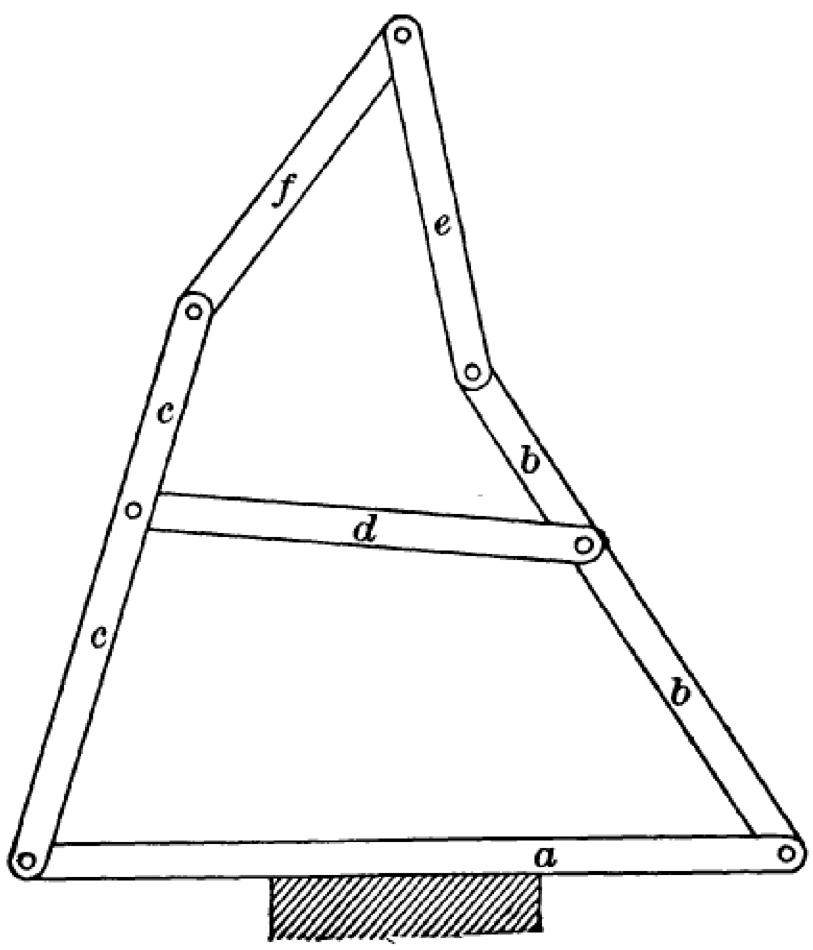

FIG. 4.

ba, *bd*, *be*, while *c* has upon it one element of each of the pairs *ca*, *cd*, *cf*.

It is worth while noticing that if the link *d* were removed the chain would no longer be a closed one. The particular mechanism shown in Fig. 4 will be again referred to.*

In the last two figures the links have been represented by straight bars. From a kinematic point of view, however, the mechanisms or chains would have been unchanged if the form of the bars had been altered in any way, always supposing that the axes of the joints remain parallel and at the same distance apart, and that the forms of the links are not such as to cause fouling or interference while the mechanism is in motion. It is evident that these remarks apply generally, and we may say that, as a rule,

* See Fig. 59.

the form or shape of a link in a chain is not of importance in kinematics, so long as the form adopted does not render impossible any portion of the required movement of the link. Questions of form and shape fall within the province of the science of Machine Design.

We have already seen that in discussing whether a kinematic chain is or is not equivalent to a mechanism, we suppose one link to be fixed, and we then proceed to determine whether the chain is closed or not; a closed chain having one link fixed being regarded as a mechanism.

The choice of the fixed link is left open, and by selecting different links of a kinematic chain different mechanisms are generally obtained. Thus, in general, from a given kinematic chain we may derive as many mechanisms as the chain has links. These mechanisms are called the *inversions* of the original chain, and, as in the case of the inversion of pairs, the exchange of one fixed link for another is known as the *inversion of the chain.* Many examples of such inversion will be met with in the following chapters.

5. Motion and Position in a Plane.—Kinematics is simply the science of pure motion, as is indeed indicated by its name (from κίνημα, motion), first suggested by Ampère. Some of the simpler propositions of pure kinematics will be given here before explaining their application in the special case of the kinematics of machines. They are based on geometrical principles, since they deal with the ideas of position and space. But it will be at once seen that the introduction of the ideas of time, and consequently of velocity and acceleration, extends the scope of the science of kinematics considerably beyond the limits of pure geometry.

Two chief classes of problems arise, the first dealing with the position and motion of a particle, and the second treating of similar questions relating to rigid bodies. The motion of non-rigid bodies is of course of a far more complex nature, and only a few elementary cases will fall within the limits of this work. Indeed the motion of such bodies cannot be investigated apart from the forces acting on them, and its consideration falls within the province of Kinetics, rather than within that of Kinematics.

Motion is defined as change of position, and is known if the position of the point or body considered is known for every instant. The position of a point or of a body can only be defined in relation to another point or body (as the case may be) whose position is fixed, or, in other words, whose change of position is neglected. Position (and therefore motion) is then purely relative. When we speak of a mountain being ten thousand feet in height, we are referring the position of its summit to an arbitrary datum surface, that of mean sea-level. In stating the position of a point or body (a body being equivalent to a system of points) we must then refer to some other point or body, and in considering the motion of a point or system of points, such motion can only be imagined with reference to a second point or system of points, supposed to be fixed.

In the case of plane motion, this reference system is usually taken to be the surface on which is drawn the diagram representing the motion of the body considered. In order to define the plane motion of a plane figure, with regard to a plane, it is sufficient to know the motion of any two points in the figure with reference to the plane. The truth of this will be seen by considering that if the motion of one point only were known, we should still be ignorant of any rotation the figure might have about an axis perpendicular to the plane. The knowledge of another point's motion, however, defines such rotation.

In most cases, problems arising in the kinematic study of machines are found to involve the consideration of *Plane Motion* only.

A rigid body having Plane Motion moves in such a way that all planes originally parallel to a certain fixed plane (that of motion) remain parallel thereto during the whole movement of the body, while any point whatever in the body moves in a plane either parallel to or coincident with the plane of motion.

A body moving in this manner will in fact have no motion of translation in a direction normal to the plane of motion, and the position of the body with respect to the plane of motion will agree exactly with the position of its projection on the plane of motion. Hence in considering the plane motion of rigid bodies, we need deal only with the kinematics of plane figures, and all propositions relating to the plane motion of plane figures will be applicable to that of rigid bodies.

It is not, in general, so necessary to trace out the whole motion of a body as to know what is its *instantaneous motion* at some given stage of its movement. By this term is meant the change of position executed by the body in a very small period of time. The manner in which these small changes of position follow one another must now be considered for the case of plane motion.

In Figure 5, let AB, $A'B'$, represent two successive positions of a plane figure (as defined by the position of two

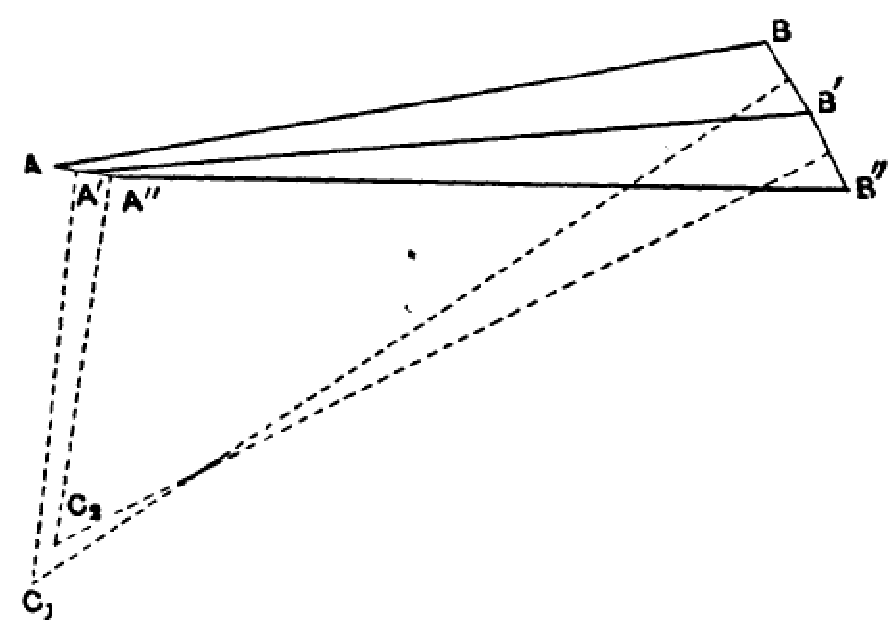

FIG. 5.

points A and B in it) at the beginning and end of an interval of time which is very small as compared with the whole period of motion.

Join AA', BB', and bisect the lines AA', BB', by straight lines perpendicular to AA', BB', and intersecting at C_1.

Then it is plain that $C_1A = C_1A'$ and $C_1B = C_1B'$, and if the point A had described a very small circular arc with centre C_1, its new position would have been A', and its path would have been indistinguishable from the line AA'. The actual infinitesimally small change of position of the point A is therefore the same as if it had been rotated in the plane of motion around an axis perpendicular to the plane and passing through C_1, and similarly for the point B. Thus, knowing the change of position of two points in the rigid figure considered, we say that the actual instantaneous motion of the body AB has been equivalent to a *virtual rotation* about the centre C_1. During the next instant the instantaneous motion may be around some other point C_2 indefinitely near to C_1, and so on. The point C_2 corresponds to the movement from $A'B'$ to $A''B''$. Thus to every part of the motion of AB, with regard to the plane, there corresponds a certain point C in the plane, about which an equivalent virtual rotation has taken place. Such points, as $C_1, C_2 \ldots$, are called the *instantaneous* or *virtual centres* of AB with regard to the plane. The locus of C_1, or the curve described by the point C on the plane, is known as the *centrode* of AB with regard to the plane, and, in general, it forms a continuous curve.

In the case of a rigid body having plane motion, it would be more correct to consider the equivalent rotation as taking place about a *virtual axis* (perpendicular to the plane of motion) of which the points $C_1, C_2 \ldots$ are the successive traces on the plane of motion. Such a virtual axis would then describe a surface in space, this surface being known as the *axode* of the body with regard to the plane of motion. For most cases of plane motion, however, we are content to simplify matters by considering the centrode instead of the axode. We shall see later that in more complex forms of motion the axode becomes of great kinematic importance. It is in every case what is called a ruled surface, i.e., a surface described by successive positions of a straight line in space.

Referring again to the plane motion of the figure AB (Fig. 5), let us inquire what happens if our construction fails. This will occur if the bisectors of the lines AA' and BB' are parallel, in which case the successive positions of AB are also parallel to one another, and the motion of the body, or of the figure it represents, is one of simple translation in a straight line. The virtual centre for such motion as this is then at an infinite distance, and we may regard any plane motion of translation in a straight line as equivalent to a rotation about an infinitely distant centre. Again, suppose that one of our reference points A does not change its position at all. It is easily seen that AB has now simple rotation about A, and during the continuation of this motion we have no longer a virtual but a permanent centre. It may happen that the lines bisecting AA' and BB' are coincident. A little consideration will show that in this case, since the triangles ABC_1 and $A'B'C_1$ must be equal in all respects, the point C_1 is at the intersection of AB and $A'B'$, produced if necessary; as before, a simple rotation about C_1 would suffice to move AB into the new position $A'B'$.

It is thus shown that in every case *the motion of a plane figure in a plane may be regarded as equivalent to a simple rotation about some actual or virtual centre*, whose position in the plane will be fixed in the case of simple rotation, or will be at an infinite distance in the case of simple translation. Such a virtual centre, however, is in general neither fixed, nor at an infinite distance, but changes its position as the body moves, and its locus in the plane is the centrode of the body with reference to the plane. Note that only rigid bodies or figures can have centrodes, for we assume that the position of our reference-line AB in the figure or body remains unchanged throughout the motion, and we represent a rigid body by the line joining the two points in question.

It has thus been seen that the centrode of a body with regard to the plane of motion is a curve described on that plane by the virtual centre of the body. Let us now consider the relative motion of two bodies in a plane. Instead of supposing that the virtual centre M (Fig. 6) of the first body AB traces its centrode on the plane of motion, imagine that the curve is marked on a sheet of paper or surface rigidly attached to the second body CD, and that the body CD is fixed. The point M is then the one point common to the two bodies AB and CD *at which there is no relative motion*, for the only possible relative motion would be rotation about the point M, a motion which is non-existent as far as a point is concerned. M is the virtual centre of AB relatively to CD, but evidently it might equally well be called the virtual centre of CD relatively to AB. Next suppose that AB is fixed, and let CD have exactly the same relative motion as before. At the instant when the relative positions of AB and CD are the same as those just considered, the virtual centre will be the same point M, but it may now be supposed to describe its centrode on the body AB, and not on CD. This centrode (that of CD relatively to AB) will not be the same curve as that described before, although they must have one point M in common at any instant. It is evident, therefore, that the two centrodes corresponding to the relative motion of two bodies always touch at a point, which is the virtual centre for the instant considered, and we may represent such relative motion by the rolling on one another of a pair of centrodes. Further, we shall find that from the form of these centrodes we can determine the relative motion of the two bodies.

To make this clearer, the two cases of motion are represented in Fig. 6. AB and CD represent the original positions of the two bodies, and, CD remaining fixed, A_1B_1, $A_2B_2 \ldots A_5B_5$ represent successive positions of AB, the motion from AB to A_1B_1 corresponding to a rotation about a virtual centre M_1, and so on. The curve $M_1M_2 \ldots M_5$ is then the centrode of AB with regard to CD.

Next we have plotted the positions C_1D_1, $C_2D_2 \ldots C_5D_5$ which CD would occupy, supposing that the relative motion were the same as before, but that AB now remained fixed. For example, in the figure C_3D_3 has the same position relative to AB that A_3B_3 has to CD, and so on for all the other positions. We now find the series of virtual centres M_1,

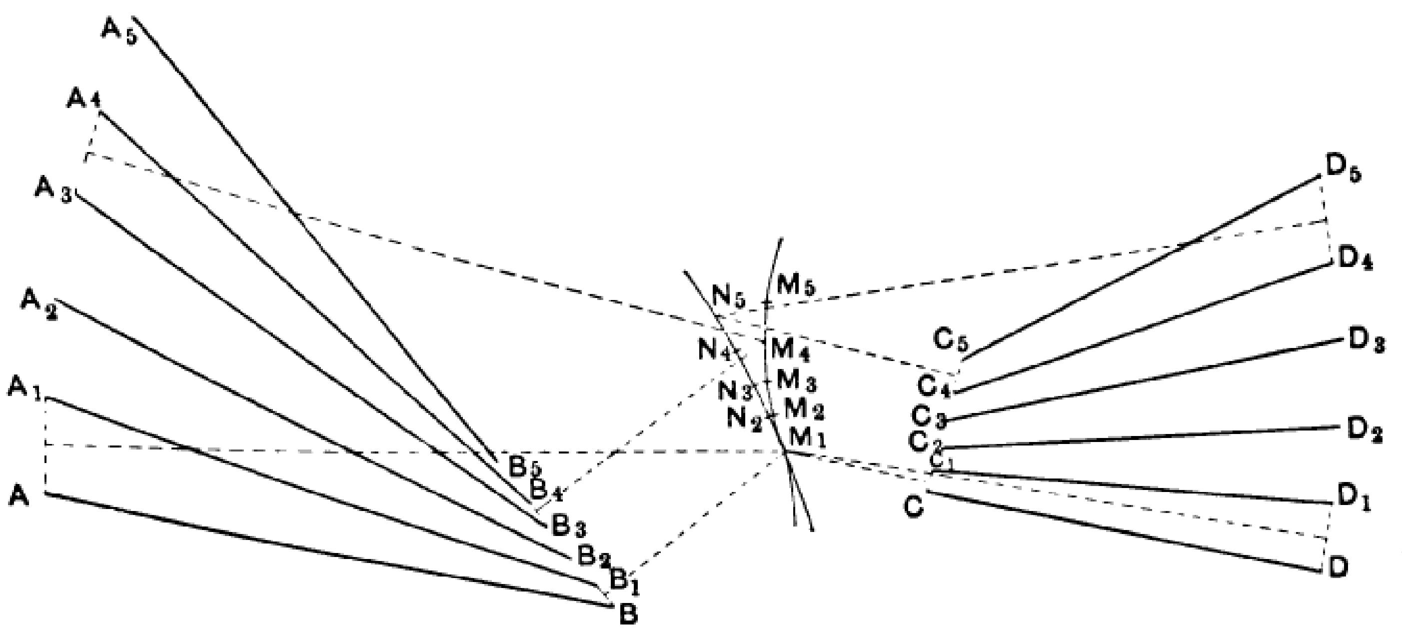

FIG. 6.

$N_2 \ldots N_5$ by the construction previously explained, and see that these centres lie on another curve touching the first at M_1 and forming the centrode of CD with regard to AB.

Remembering that this curve is attached to, or rather described on, the body represented by AB, suppose that CD remains fixed, while AB (with the centrode attached) moves from AB to A_1B_1, i.e., AB rotates instantaneously about M_1. If the movement is imitated by tracing AB and the curve $M_1, N_2 \ldots N_5$ on paper and placing AB in the position A_2B_2, it will be found that N_2 coincides with M_2. When AB is at A_3B_3, M_3 and N_3 coincide, and so on. Such successive coincidences can only occur if the curve M_1N_5 rolls on the curve M_1M_5.

In the same way if we trace CD and the curve M_1M_5, and let CD occupy its successive positions, we find that the points coincide as before, the curve M_1M_5 now rolling on M_1N_5.

Thus the given relative motion of AB and CD, through the successive positions shown on the figure, is represented by the rolling on one another of two curves, the pair of centrodes of the two bodies.

The reader is strongly recommended to satisfy himself of the correctness of the above statements by actually drawing a pair of bodies, and their centrodes for a given case of relative motion. Great care and accuracy in drawing are necessary in order to obtain correct positions for the virtual centres.

We have now discussed the case of the relative motion of two bodies in a plane, and have seen that their virtual centre describes a pair of curves, namely the centrodes, each being traced on one of the two bodies.

Suppose next that we have three bodies, represented, as before, by plane figures, and having any kind of relative plane motion. The three bodies will evidently have three virtual centres, while four bodies would have six, and so on; in fact, a kinematic chain having plane motion and consisting of n links will have $\frac{n(n-1)}{2}$ virtual centres connected with it, for it will easily be seen that the number of virtual centres must be that of the combinations of n things taken two at a time.

On examination of any particular case we shall see that the various virtual centres in a mechanism having plane motion are arranged in threes, each three lying in a straight line, whatever be the position of the mechanism.

The proof of this statement is as follows: Consider any three of the bodies, or links forming the kinematic chain or mechanism, and let us call them a, b, c. Denoting the virtual centre of a with regard to b by O_{ab} and remembering that this is the same point as the virtual centre of b with regard to a, we have for the three bodies considered the three virtual centres O_{ab}, O_{ac}, O_{bc}. First consider b as being fixed. Then with regard to the point O_{ab} any point in a has a simple motion of rotation, so that, for example, the point O_{ac} is moving instantaneously and relatively to b in a direction at right angles to the line $O_{ac} \ldots O_{bc}$.

Again, with regard to the point O_{bc}, any point in c, such as O_{ac}, must be moving instantaneously and relatively to b in a direction at right angles to the line $O_{ac} \ldots O_{bc}$.

Thus the point O_{ac}, regarded as a point in a, is moving in a line perpendicular to $O_{ac} \ldots O_{ab}$; while if regarded as a point in c, O_{ac} moves in a line perpendicular to $O_{ac} \ldots O_{bc}$, b being regarded as fixed in each case. O_{ac} cannot have two separate directions of instantaneous movement at the same instant, hence the lines $O_{ac} \ldots O_{ab}$ and $O_{ac} \ldots O_{bc}$ are both perpendicular to the same line. They cannot be

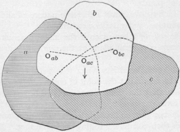

FIG. 7.

parallel, since they both pass through O_{ac}, and they therefore coincide in direction, i.e., the points O_{ab}, O_{ac}, O_{bc} lie on one straight line.

The position of the virtual centres in various mechanisms will be studied when we consider the relative velocities of their different parts. In many instances the proposition just given is of great assistance in determining the positions of the virtual centres in a mechanism.

6. Non-plane Motion.—In the majority of cases it will be found that the relative motions of the parts of machines are plane motions, either of rotation or translation, or both combined. Such motions can be studied geometrically by the method indicated in the preceding section. It is possible (as will be seen later) to have a lower pair, in which the motion is non-plane. A somewhat limited number of cases of higher pairing also occur in which the motion is non-plane.

In every instance, however, in a closed pair, we have seen that there must be continuous contact of the surfaces, and it follows that the most general possible relative motion of two parts of a mechanism is represented by the motion of one rigid body continuously touching another at a point or series of points.

Any such motion must be of the nature of sliding, rolling, or spinning, separately or combined.

Simple rolling takes place if the instantaneous axis lies in the common tangent plane at the point of instantaneous contact.

Simple spinning exists when the instantaneous axis is the common normal at the point of contact.

Suppose that the relative motion is such that the instantaneous axis passes through the point of contact, and is neither in nor perpendicular to the tangent plane. The motion is then combined rolling and spinning. If the instantaneous axis does not pass through the point of contact, the rolling and spinning will further be combined with a sliding motion.

We have a familiar example of combined rolling and sliding in the mutual action of a pair of teeth in an ordinary spur-wheel; the motion of the balls in a bicycle bearing, again, is a case of combined rolling and spinning.

The links of a certain class of mechanism are found to have such motions that their instantaneous axes all pass through a fixed point, while each portion of every link remains at its own constant distance from that point. Such motion is called *spheric motion*, because any given point on a link must be always on the surface of a sphere described about the fixed point as centre. It is evident that the most general case of spheric motion is that of a rigid body of which one point is fixed, and any kind of spheric motion can be made up by combining spins about axes passing through the fixed point. Plane motion may be looked upon as a particular case of spheric motion, in which the radius of the spheres is infinitely large.

7. **Freedom and Constraint.** — We have seen that the essential feature of a kinematic pair is the mutual constraint due to the forms of the two elements of which the pair is composed. Before considering the ways in which constraint or closure is actually applied it will be well to examine briefly the conditions on which the freedom of movement of a rigid body depends.

The most general motion of a free rigid body may be looked upon as being a combination of three independent rotations about three rectangular axes, with three independent motions of translation along those axes. Such a body may then be said to have *six degrees of freedom*, one of which is taken away (or one degree of constraint is imposed) when any one of these six modes of movement is rendered impossible. Suppose that the free rigid body is forced to touch a smooth fixed surface at one point, one degree of freedom is lost, for no translation can take place in a direction normal to the tangent plane to the surface at the points of contact. The three motions of rotation, however, still remain possible, and so does motion of translation in any direction parallel to the tangent plane at the point of contact. A second point of restraint may be arranged so as to prevent one motion of rotation, or a second motion of translation, according to its position with regard to the first point of restraint and with regard to the form of the body. A third point of restraint causes the body to lose a third degree of freedom, and, finally, it will be found that all six degrees of freedom are lost, and the position of the body is fixed if six of its points are made to rest on six portions of the surface of the smooth fixed body, and if these portions are properly formed and placed.*

It may be shown that in general six conditions are required to completely determine the position of a rigid body, or, expressing the same thing in another way, six coordinates specify the position of one rigid body relatively to another, considered to be fixed.

The definitions of a closed pair or of a closed chain given in §§ 2 and 3 thus mean that any element or link in a closed pair or chain may have only one degree of freedom as referred to the fixed element or link.

Consider, for example, a screw turning in a fixed nut, like the screw of a micrometer gauge. The position of such a screw is determined exactly if an arm attached to its head is forced to remain in contact with a fixed stop on the body of the gauge, and we say, therefore, that such a screw has only one degree of freedom, inasmuch as its position is fixed by one point of constraint. The motion of a screw in its nut, a motion of translation accompanied by a definite and proportional motion of rotation whose axis is the direction of translation, is the most general kind of motion that can be possessed by a body having only one degree of freedom.

The reader will notice that in two special cases, namely, when the pitch of the screw is infinite, and when the pitch is zero, the twisting motion of the nut becomes a mere translation or a mere rotation, both being specially important as plane motion involving one degree of freedom.

In a similar way such a body as the connecting-rod of a direct-acting steam-engine is said to have constrained motion, having only one degree of freedom. The only possible motion at any instant for a given point on the rod is that of rotation about a certain virtual axis parallel to the axis of the crank-shaft.

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* See § 60, Chapter VII.

Such a contrivance as a ball-and-socket joint cannot be regarded as a closed pair, for the ball has three degrees of freedom with regard to the socket. The ball has one point fixed, its centre, thus rendering all motion of translation impossible, and causing three degrees of constraint. The socket in fact might be replaced by three pairs of points touching the sphere at the ends of three diameters, each pair of points corresponding to one degree of constraint.

Further examples may easily be imagined; the method of determining the conditions as to freedom and constraint in any particular case will be evident from the instances just given.*

8. Elements and Pairs in Rigid Links.—It has been pointed out that the pairs of elements formed on the links of which a mechanism is made up are of two kinds, namely, *lower pairs*, in which the elements are in contact with each other over the whole or part of the area of certain surfaces, and *higher pairs*, in which such contact occurs only at certain points or along lines of points.

In those portions of machines which are rigid the elements must have forms which can be readily produced by the ordinary processes of the workshop. Accordingly we find that their shapes are such as can be formed either in the lathe or the milling-machine, or by one of the many machine tools in which the cutting-tool describes a straight line with reference to the work. The rigid elements forming the closed pairs in machines therefore have in general for their working surfaces either surfaces of revolution, plane surfaces, or screw surfaces.

From the definition of *Lower Pairs* it is also plain that the forms of their elements must be such as to fit one another not only in one position, but in any position they may take up during their relative motion. It is plain also that two

* The reader may refer to Thomson & Tait, Natural Philosophy, Part I, Sections 195–201; also Tait, Enc. Brit., art. Mechanics.

surfaces of revolution, the one full and the other hollow, will fulfil this condition, and if properly formed, so as to prevent any sliding along the axis of revolution, will constitute a closed lower pair in which either element can only have constrained motion relatively to the other. Such pairs are called shortly *turning pairs*, and Fig. 8 represents two bodies, a and b, so shaped as to form such a turning pair. The body b is partly cut away, to show more clearly the outline of a.

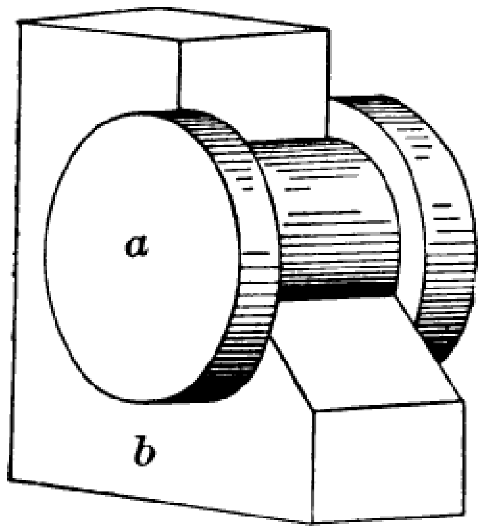

FIG. 8.

The same condition (of fitting each other in any position) obtains in the case of a screw of uniform pitch and its nut. The relative motion is also constrained, as has already been stated, and consists of a motion of rotation around the axis of the screw, combined in a constant ratio with a motion of translation along that axis. Such a pair of screw surfaces forms a *screw-pair*.*

In general a lower pair formed by two cylindrical or prismatic surfaces will have constrained relative motion, because it will only be possible to give one body a motion of translation along the generating lines of the prism or cylinder relatively to the other body. If, however, the forms are circular cylinders, which are, of course, surfaces of revolution, then indefinite turning also is possible, the motion ceases to be constrained, and the pair is no longer closed. A pair of cylindrical or prismatic surfaces for which sliding only is possible is called a *sliding pair* (see Fig. 9).

On examination it will be found that pairs of conical and other forms of surfaces generated by straight lines do not fulfil the conditions of continuous fitting or contact during motion, unless they are at the same time surfaces

* See Chapter XI.

of revolution. Non-cylindrical ruled surfaces in machines therefore have usually to take part in *higher pairing*.

The three classes of lower pairs just discussed are then the only ones found in the rigid portions of machines. Examples of each kind will present themselves on examining a few simple machines, and the means of constraint should be noticed in each case. For instance, in a shaft-

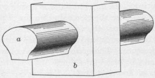

FIG. 9.

journal, endways motion or sliding of the shaft in its bearing is prevented either by making the diameter of the journal smaller than that of the adjoining portions of the shaft, or by securing collars on either side of the bearing.

All the forms of ruled surfaces mentioned above, and occasionally plane surfaces, surfaces of revolution, or screw surfaces, are found as portions of higher pairs, as well as of lower pairs.

A simple arrangement of higher pairing can frequently be used to give motion of a kind which could only be otherwise obtained by a complex chain of lower pairs.

It is important to notice that higher pairs give relative motion of a much more complex kind than is attainable by the use of lower pairing. This fact is pointed out by Burmester,* and is expressed if we say that supposing *a*

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* Lehrbuch der Kinematik, §§ 114, 116.

and b are two elements of a closed pair, and if a point A in a describes the same curve on b as a point B in b (originally coinciding with A) describes on a, then the pair is a lower pair. If, on the other hand, A describes on b a line or curve different from that described by B on a, we have a case of higher pairing. Thus in the case of a lower pair no alteration of the relative motion occurs whether we consider one or the other of the elements as being the fixed one.

Lower pairing is the more important from a constructive point of view, because the elements of a lower pair have a simpler relative motion, they are able to resist wear when transmitting heavy loads, and they can easily be made tight under fluid pressure. These are properties not possessed by higher pairs.

9. Pairing of Non-rigid Links.—Passing on to the pairing of *non-rigid links* in mechanisms, it is found that these links may be classed under the following heads:

(1) Flexible bodies, such as ropes, belts, or chains. These are almost invariably paired with cylindrical surfaces on to or from which they unwrap or wrap themselves. Such pairing may be called *tension pairing*, since the rope, belt, or chain is necessarily in tension.

(2) Pressure links, which continually exert pressure on the elements with which they pair. These links generally consist of portions of fluid, such as air, steam, or water, and pair with the interior of the vessels containing them. Such a pair is known as a *pressure pair*.

Springs often form most important portions of mechanisms and machines. They may be arranged so as to be in tension or in compression, and the resulting pairs may be said to be tension or pressure pairs, as the case may be.

Actually all machine parts are elastic and so act to a certain minute extent as springs, but in kinematics we neglect all small changes of form, and consider such pieces as being rigid, classing under the head of springs only those portions of machines whose elastic deformations under load are considerable in extent when compared with the proper motions of the other machine parts or links with which they pair.

Non-rigid links will be considered at greater length subsequently.

10. Classification of Mechanisms. — In attempting to classify mechanisms, which are made up of various kinds of links and involve so many kinds of pairing, we are impressed with the magnitude and complexity of the task. It may be said, in fact, that up to the present no wholly satisfactory kind of machine classification has been proposed. Some account of what has been done in this direction will be found in Chapter XIII; for present purposes it will be sufficient to consider mechanisms under three heads.

(1) Those involving only plane motion. These may be called shortly *Plane Mechanisms*, and form by far the most mportant and numerous class.

(2) Mechanisms involving spheric motion, or, more briefly, *Spheric Mechanisms.*

(3) Chains the relative motion of whose links is neither plane nor spheric, but of greater complexity.

It is, however, to be understood that a mechanism of the third kind may contain certain links whose motion is plane or spheric, while any of them may include examples of both lower and higher pairing.

A well-known instance of a spheric mechanism is Hooke's joint, the characteristic property of such chains being that the axes of the turning pairs they contain meet in a point. In the third class the most common examples are screw mechanisms.

There is another method of classifying machines according to their geometrical properties, and according to the methods necessary for determining the various virtual centres of their links. Following this system, we should say that mechanisms of the *First Order* are those in which, having given the relative position of *any two* links, the positions of all the other links may be found by geometrical construction of straight lines and circles. From this it follows that in such mechanisms, having given the whole mechanism in one position, we can find geometrically all its other possible positions, and the virtual centre of each link relatively to every other. Mechanisms not possessing these properties belong to higher orders, and are of comparatively infrequent occurrence.