Unit-6
Statistical Quality Control and Acceptance Sampling

Contents: Importance of statistical method in quality control, ND curve, Different types of control charts(X Bar, R, P and C charts), their constructions, Interpretation and applications, Basic concept of sampling inspection, Operating characteristic curves, Conflicting interests of consumer and producer, Producer and consumers risks, Single and double sampling plans.

Introduction:

Statistical quality control relies on statistics and data reports to assess quality and is therefore an efficient means to evaluate a manufactured product. SQC helps maintain the consistency of how a product is made. SQC methods can include cause-and-effect analysis, check/tally sheets, histograms, Pareto and scatter analyses, data stratification, defect maps events logs. One important method of statistical quality control is acceptance sampling. In acceptance sampling, a sample of a product is randomly taken to determine whether or not to continue making the product.

Statistical Quality Control

As a response to the impracticality of inspecting every production item, methods involving sampling techniques were suggested. The behaviour of samples as an indicator of the behavior of the entire population has a strong statistical body of theory to support it.

A landmark in the development of statistical quality control came in 1924 as a result of the work of Dr. Walter Shewhart during his employment at Bell Telephone Laboratories. He recognized that in a manufacturing process there will always be variation in the resulting products. He also recognized that this variation can be understood, monitored, and controlled by statistical procedures. Shewhart developed a simple graphical technique - the control chart - for determining if product variation is within acceptable limits. In this case the production process is said to be ‘in control’ and control charts can indicate when to leave things alone or when to adjust or change a production process. In the latter cases the production process is said to be ‘out of control.’ Control charts can be used (importantly) at different points within the production process.

The aim of statistical process control is to ensure that a given manufacturing process is as stable(in control) as possible and that the process operates around stated values for the product withas little variability as possible. In short, the aim is the reduction of variability to ensure thateach product is of a high a quality as possible. Statistical process control is usually thought of asa toolbox whose contents may be applied to solve production-related quality control problems.

Sources of Variation: Common and Assignable Causes

If you look at bottles of a soft drink in a grocery store, you will notice that no twobottles are filled to exactly the same level. Some are filled slightly higher and someslightly lower. Similarly, if you look at blueberry muffins in a bakery, you will noticethat some are slightly larger than others and some have more blueberries than others.These types of differences are completely normal. No two products are exactly alikebecause of slight differences in materials, workers, machines, tools, and other factors.These are called **common, or random, causes of variation.** Common causes of variationare based on random causes that we cannot identify. These types of variation areunavoidable and are due to slight differences in processing.

An important task in quality control is to find out the range of natural randomvariation in a process. For example, if the average bottle of a soft drink called CocoaFizz contains 16 ounces of liquid, we may determine that the amount of natural variationis between 15.8 and 16.2 ounces. If this were the case, we would monitor theproduction process to make sure that the amount stays within this range. If productiongoes out of this range—bottles are found to contain on average 15.6 ouncesthis would lead us to believe that there is a problem with the process because the variationis greater than the natural random variation.

The second type of variation that can be observed involves variations where thecauses can be precisely identified and eliminated. These are called **assignable causesof variation.** Examples of this type of variation are poor quality in raw materials, anemployee who needs more training, or a machine in need of repair. In each of theseexamples the problem can be identified and corrected. Also, if the problem is allowedto persist, it will continue to create a problem in the quality of the product. In the exampleof the soft drink bottling operation, bottles filled with 15.6 ounces of liquidwould signal a problem. The machine may need to be readjusted. This would be anassignable cause of variation. We can assign the variation to a particular cause (machineneeds to be readjusted) and we can correct the problem (readjust the machine).

Control Charts:

A **control chart** (also called process chart or quality control chart) is a graph thatshows whether a sample of data falls within the common or normal range of variation.A control chart has upper and lower control limits that separate common fromassignable causes of variation. The common range of variation is defined by the use of control chart limits. We say that a process is **out of control** when a plot of data revealsthat one or more samples fall outside the control limits. Figure shows a control chart for the Cocoa Fizz bottling operation. The *x* axisrepresents samples (#1, #2, #3, etc.) taken from the process over time. The *y* axis representsthe quality characteristic that is being monitored (ounces of liquid). The centerline (CL) of the control chart is the mean, or average, of the quality characteristicthat is being measured. In figure the mean is 16 ounces. The upper control limit(UCL) is the maximum acceptable variation from the mean for a process that is in astate of control. Similarly, the lower control limit (LCL) is the minimum acceptablevariation from the mean for a process that is in a state of control. In our example, theupper and lower control limits are 16.2 and 15.8 ounces, respectively. You can see thatif a sample of observations falls outside the control limits we need to look for assignablecauses.

The upper and lower control limits on a control chart are usually set at ±3 standarddeviations from the mean. If we assume that the data exhibit a normal distribution,these control limits will capture 99.74 percent of the normal variation. Controllimits can be set at ±2 standard deviations from the mean. In that case, control limitswould capture 95.44 percent of the values. Figure shows the percentage of valuesthat fall within a particular range of standard deviation.

Looking at Figure, we can conclude that observations that fall outside the set rangerepresent assignable causes of variation. However, there is a small probability that a valuethat falls outside the limits is still due to normal variation. This is called Type I error, withthe error being the chance of concluding that there are assignable causes of variationwhen only normal variation exists. Another name for this is alpha risk (α), where alpharefers to the sum of the probabilities in both tails of the distribution that falls outside theconfidence limits. The chance of this happening is given by the percentage or probabilityrepresented by the shaded areas of Figure. For limits of ±3 standard deviations fromthe mean, the probability of a Type I error is 0.26% (100% - 99.74%), whereas for limitsof ±2 standard deviations it is 4.56% (100% - 95.44%).

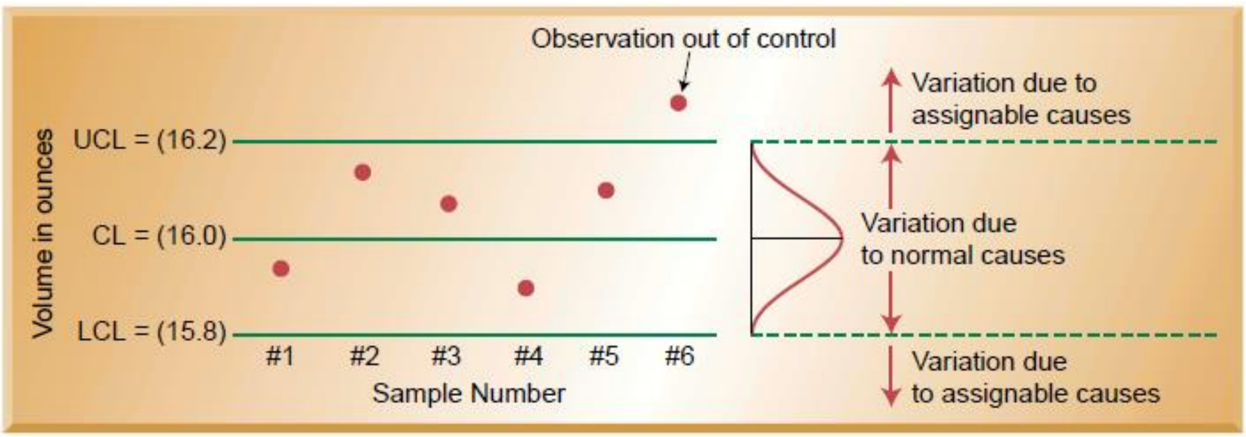


Fig 6.1. Quality control chart for Cocoa Fizz

ACCEPTANCE SAMPLING

Acceptance sampling, the third branch of statistical quality control, refers to the process of randomly inspecting a certain number of items from a lot or batch in order to decide whether to accept or reject the entire batch. What makes acceptance sampling different from statistical process control is that acceptance sampling is performed either before or after the process, rather than during the process. Acceptance sampling before the process involves sampling materials received from a supplier, such as randomly inspecting crates of fruit that will be used in a restaurant, boxes of glass dishes that will be sold in a department store, or metal castings that will be used in a machine shop. Sampling after the process involves sampling finished items that are to be shipped either to a customer or to a distribution center. Examples include randomly testing a certain number of computers from a batch to make sure they meet operational requirements, and randomly inspecting snowboards to make sure that they are not defective.

You may be wondering why we would only inspect some items in the lot and not the entire lot. Acceptance sampling is used when inspecting every item is not physically possible or would be overly expensive, or when inspecting a large number of items would lead to errors due to worker fatigue. This last concern is especially important when a large number of items are processed in a short period of time. Another example of when acceptance sampling would be used is in destructive testing, such as testing vehicles for crash testing. Obviously, in these cases it would not be helpful to test every item! However, 100 percent inspection does make sense if the cost of inspecting an item is less than the cost of passing on a defective item.

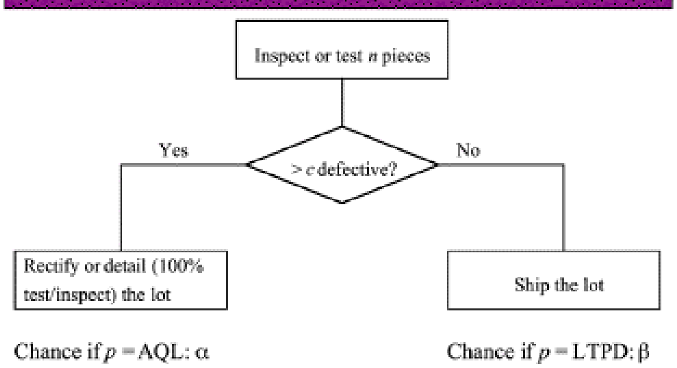
As you will see in this section, the goal of acceptance sampling is to determine the criteria for acceptance or rejection based on the size of the lot, the size of the sample, and the level of confidence we wish to attain. Acceptance sampling can be used for both attribute and variable measures, though it is most commonly used for attributes. In this section we will look at the different types of sampling plans and at ways to evaluate how well sampling plans discriminate between good and bad lots.

Sampling Plans

A **sampling plan** is a plan for acceptance sampling that precisely specifies the parameters of the sampling process and the acceptance/rejection criteria. The variables to be specified include the size of the lot (*N*), the size of the sample inspected from the lot (*n*), the number of defects above which a lot is rejected (*c*), and the number of samples that will be taken.

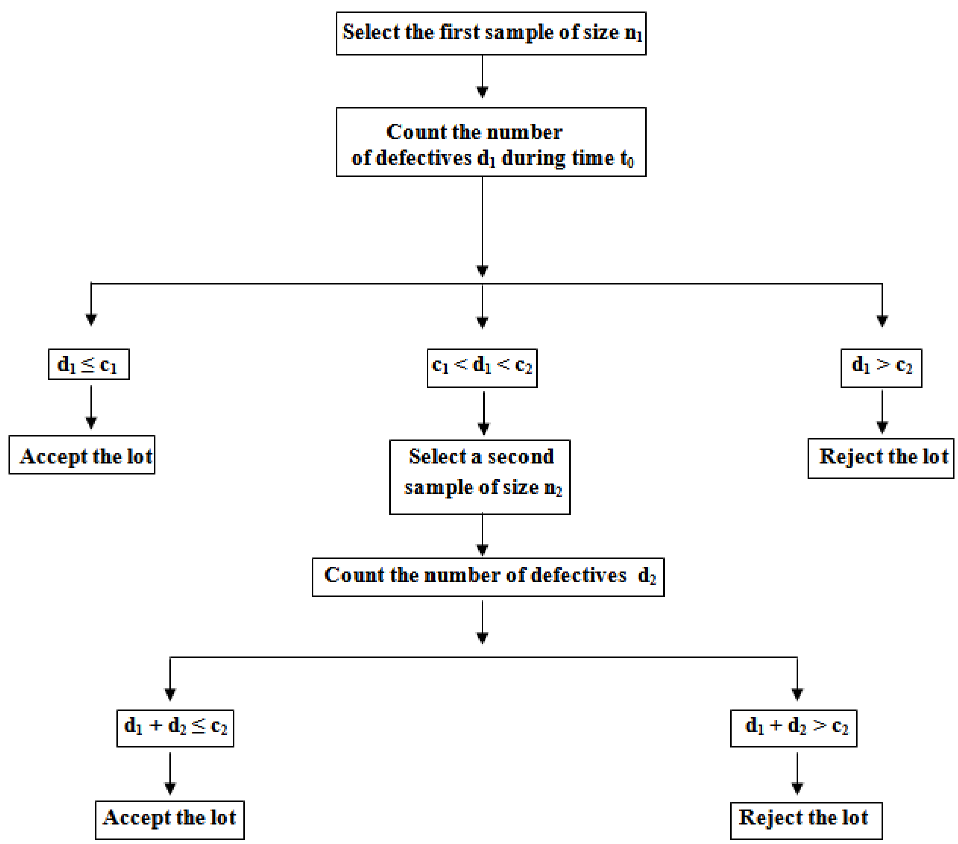
There are different types of sampling plans. Some call for **single sampling**, in which a random sample is drawn from every lot. Each item in the sample is examined and is labeled as either "good" or "bad." Depending on the number of defects or "bad" items found, the entire lot is either accepted or rejected. For example, a lot size of 50 cookies is evaluated for acceptance by randomly inspecting 10 cookies from the lot. The cookies may be inspected to make sure they are not broken or burned. If 4 or more of the 10 cookies inspected are bad, the entire lot is rejected. In this example, the lot size N = 50, the sample size n = 10, and the maximum number of defects at which a lot is accepted is c = 4. These parameters define the acceptance sampling plan.

Figure 1: Single-Sample Acceptance Sampling Procedure



Another type of acceptance sampling is called **double sampling**. This provides an opportunity to sample the lot a second time if the results of the first sample are inconclusive. In double sampling we first sample a lot of goods according to preset criteria for definite acceptance or rejection. However, if the results fall in the middle range, they are considered inconclusive and a second sample is taken. For example, a water treatment plant may sample the quality of the water ten times in random intervals throughout the day. Criteria may be set for acceptable or unacceptable water quality, such as 0.05 percent chlorine and 0.1 percent chlorine. However, a sample of water containing between 0.05 percent and 0.1 percent chlorine is inconclusive and calls for a second sample of water.

In addition to single and double-sampling plans, there are **multiple sampling plans.** Multiple sampling plans are similar to double sampling plans except that criteria are set for more than two samples. The decision as to which sampling plan to select has a great deal to do with the cost involved in sampling, the time consumed by sampling, and the cost of passing on a defective item. In general, if the cost of collecting a sample is relatively high, single sampling is preferred. An extreme example is collecting a biopsy from a hospital patient. Because the actual cost of getting the sample is high, we want to get a large sample and sample only once. The opposite is true when the cost of collecting the sample is low but the actual cost of testing is high. This may be the case with a water treatment plant, where collecting the water is inexpensive but the chemical analysis is costly. In this section we focus primarily on single sampling plans.



OC Curves:

The OC Curve is used in sampling inspection**.** It plots the probability of accepting a batch of items against the quality level of the batch.

Acceptance sampling is a method of measuring random samples of populations called “lots” of materials or products against predetermined standards. Acceptance sampling is a part of operations management or of accounting auditing and services quality supervision. It is important for industrial, but also for business purposes helping decision-making process for the purpose of quality management. Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. Acceptance sampling is a major field of statistical quality control. A typical application of acceptance sampling is as follows. A company receives a shipment of product from a vendor. This product is often a component or raw material used in the company’s manufacturing process.

A sample is taken from the lot and some quality characteristics of the units in the sample inspected. On the basis of the information in this sample a decision is made regarding lot disposition usually this decision is either to accept or to reject the lot. Sometimes we refer to this decision as lot sentencing. Accepted lots are put into productions; rejected lots may return to the vendor or may be subjected to some other lot disposition action.

While it is customary to think of acceptance sampling as a receiving inspection activity, there are other uses of sampling methods for example; frequently a manufacturer will sample and inspect its own product at various stages of production. Lots that are accepted are sent forward for further processing while rejected lots may be reworked or scrapped.

Acceptance sampling is most likely to useful in the following situation.

1. When testing is destructive.
2. When the cost of 100% inspection is extremely high.
3. When 100% inspection is not technologically feasible or would required so much calendar time that production scheduling would be seriously impacted.
4. When there are many items to be inspected and the inspection error rate is sufficiently high than 100% inspection might cause a higher percentage of defectives units to be passed than would occur with the use of sampling plan.
5. When the vendor has an excellent quality history, and some reduction in inspection from 100% is desired, but the vendor’s process capability ratio is sufficiently low to make no inspection an unsatisfactory alternative.
6. When there are potentially serious product liability risks and although the vendor’s process is satisfactory a program for continuously monitoring the product is necessary.

The products may be grouped into batches or lots or may be single pieces from a continuous operation. A random sample is selected and could be checked for various characteristics. For lots, the entire lot is accepted or rejected in the whole. The decision is based on the pre-specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test. In the case of continuous production process, a decision may be made to continue sampling or to check subsequent product 100%.

The figure shows an 'OC' (Operating Characteristic) Curve for a sample of 50 items taken from a batch of 2000 and using a critical acceptance number 'c' of 2 (the batch will be accepted if there are two or less defectives in the sample). From the curve you can see that there is about a 23% probability of accepting a batch that contains 8% of defective items.

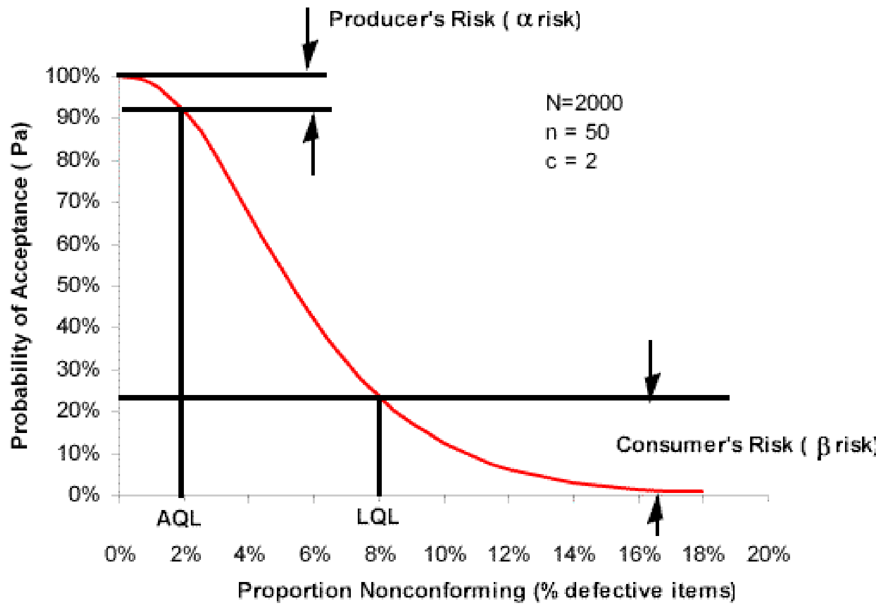


Fig. Operating Characteristic (O.C) Curve

When designing a sampling plan it is usual to decide on two points, the AQL and LQL and the associated Producer's Risk and Consumer's Risk. The necessary sample size and acceptance number for the curve to pass through these points is then calculated and hence the shape of the curve.

OC Curves are mainly associated with sampling inspection but they are also used to find the Average Run Length in control charts.

Definitions:-

Acceptable Quality Level (AQL):

AQL is defined as the maximum percentage of defects {or maximum number of defects per hundred units) that, for the purposes of sampling inspection, can be considered satisfactory. (Note: percentage defective C for (sated) probability of acceptance).

It is usual to lay down a quality level, which the customer will consider acceptable in most instances. It is common practice to set the AQL at 0.95% probability of acceptance, which means that if batches of the minimum acceptable quality are submitted, 95% will be accepted on average. There is, of course, a chance that sometimes batches of a worse quality may be accepted, as the ideal of curve is unattainable

Lot Tolerance Per Cent Defective (LTPD):

LTPD is defined as the incoming fraction defective (or number of defects per 100 units) in a lot that the consumer is willing to accept with a very small probability of occurrence. This point is shown at the lower end of the OC curve. It is often chosen for 10% consumer risk; that is, there is a 10% chance that the consumer will accept batches whose quality is worse than the acceptable level. From Fig, it can be seen that the consumer has 10% chance of accepting batches worse than 6.2% defective.

Indifference Quality (Split Risk or Point of Control):

The indifference quality is the quality level, which has an even chance of acceptance or rejection, the producer and consumer taking the same risk in the application of the plan.

Average Outgoing Quality Limit (AOQL):

The AOQL is the maximum average outgoing quality level, which, in the long run, will not become worse whatever the quality of the incoming items. It is the limit to the average quality of batches after inspection. The existence of the AOQL depends on the following conditions: 1. All rejected batches must be re-inspected 100%. 2. Every defective in the re-inspected batches must be detected. 3. Every defective item found must be replaced by a good item so that batch size remains constant. Since conditions 2 will not often be met the AOQL is an approximation; but it is a useful concept. Values of AOQL are found by multiplying together the incoming quality (per cent defective) and the proportion of batches expected to be accepted. Remember that the word 'average' is important because individual batches getting through inspection may be far worse than the AOQL and it is only when, a large number of batches is averaged that the quality cannot, in the long run, be worse than the AOQL.

Risks in Acceptance Sampling:

Neither sampling, nor 100% inspection can guarantee that every defective item in a lot shall be located. Sampling involves a risk that the sample may not adequately reflect the condition of the lot. i.e., it may not represent the lot correctly.100% inspection has the risk that monotony and excess amount of inspection work will result in inspectors missing some of the defective components. Sampling risks are of two kinds, (1) Producer's risk (2) Consumer's risk. The operating characteristics (OC) curve for a sampling plan quantifies these risks.

Producer's Risk: Assume that the lot produced is good, but unluckily the sample selected out of this lot did not represent the lot faithfully. The sample had a higher proportion of defectives than the lot as a whole. Hence the (otherwise) good lot got rejected, resulting in loss for the producer. This is known as producer's risk. Producer’s risk is designated as the alpha risk. Producer hopes to keep this risk low, say at 1 to 5% if a good lot is rejected; it is referred to as type.

Consumer's Risk: Assume that the *lot* is bad, but the sample selected out of this lot did not represent the lot truly. In other words, the sample had a higher proportion of good components than the lot as a whole. Hence the bad lot got selected, resulting in loss for the consumer. This is known as consumer's risk. Consumer's risk is designated as the Beta risk. Consumer wants to keep this risk low. If a bad lot is accepted, it is referred to as type II error. The a risk at the AQL level and the (3 risk at the LTPD level establish two points from which the sample size. n, and acceptance number, c are determined. Given these two points, the OC curve can be drawn to describe the risk characteristics of the specific sampling plan.

Characteristics of O. C. Curve:

1. The operating curve of an acceptance plan shows the ability of plan to distinguish between good and bad lots.
2. Sampling acceptance plan with same parent sample gives very definite quality protection.
3. Fixed sample size tends towards constant quality protection it is absolute size of sample relation than its relative size that determines the quality protection given by acceptance sampling plan.
4. The O.C. curves of plans with acceptance no greater than zero are superior to those of comparator plans with acceptance of zero.