# Module 2 - GEARS

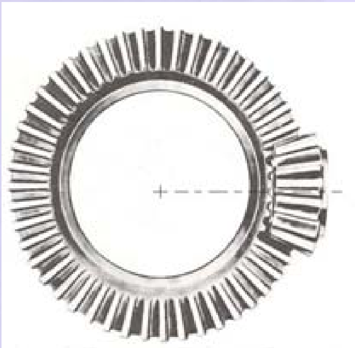
## Lecture 13 – BEVEL GEARS

### Contents

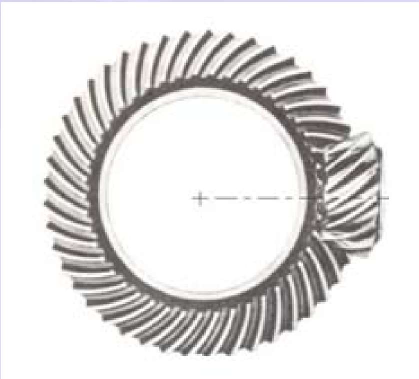


### 13.1 INTRODUCTION

**(a)**



**(b)**



**(c)**

**(d)**

**Fig.13.1 (a) Bevel gear, (b) Straight bevel gear, (c) Spiral bevel gear (d) Hypoid gear**

Bevel gears transmit power between two intersecting shafts at any angle or between non- intersecting shafts. They are classified as straight and spiral tooth bevel and hypoid gears as in Fig.13.1

## 13.2 GEOMETRY AND TERMINOLOGY

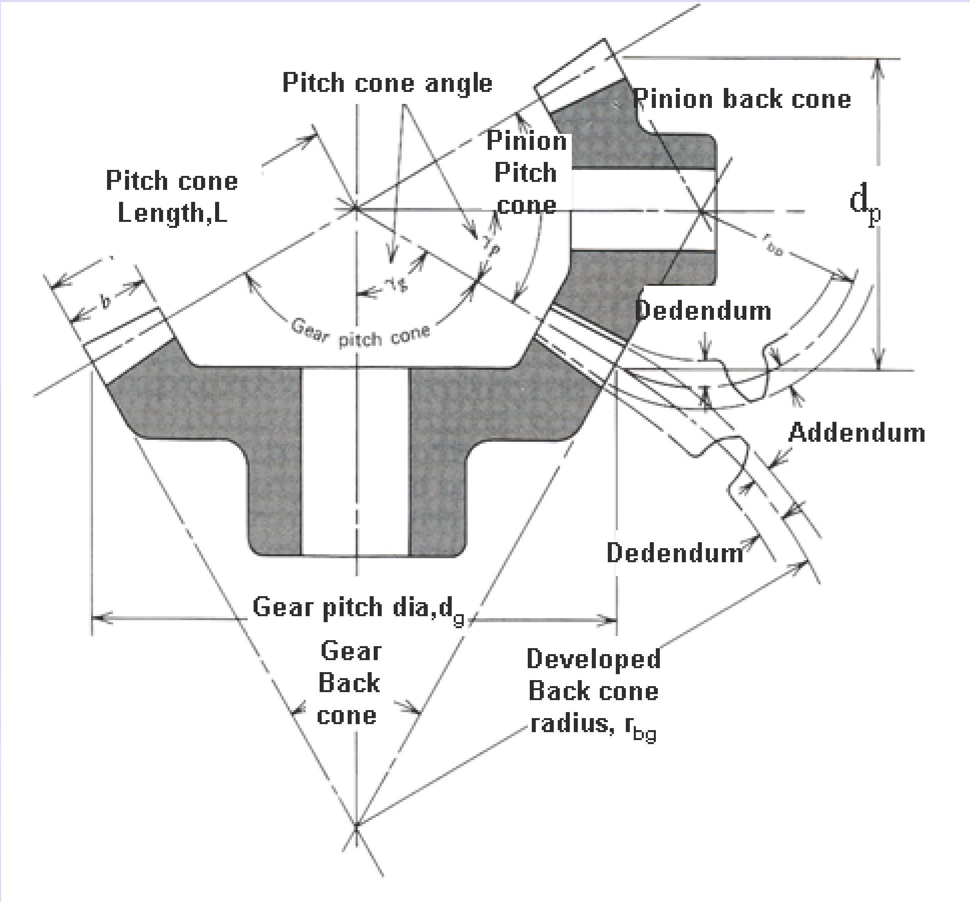



**Fig.13.2 Bevel gear in mesh**

When intersecting shafts are connected by gears, the pitch cones (analogous to the pitch cylinders of spur and helical gears) are tangent along an element, with their apexes at the intersection of the shafts as in Fig.13.2 where two bevel gears are in mesh.

The size and shape of the teeth are defined at the large end, where they intersect the back cones. Pitch cone and back cone elements are perpendicular to each other. The tooth profiles resemble those of spur gears having pitch radii equal to the developed back cone radii $r_{bg}$ and $r_{bp}$ and are shown in Fig. 13.3. which explains the nomenclatures of a bevel gear.

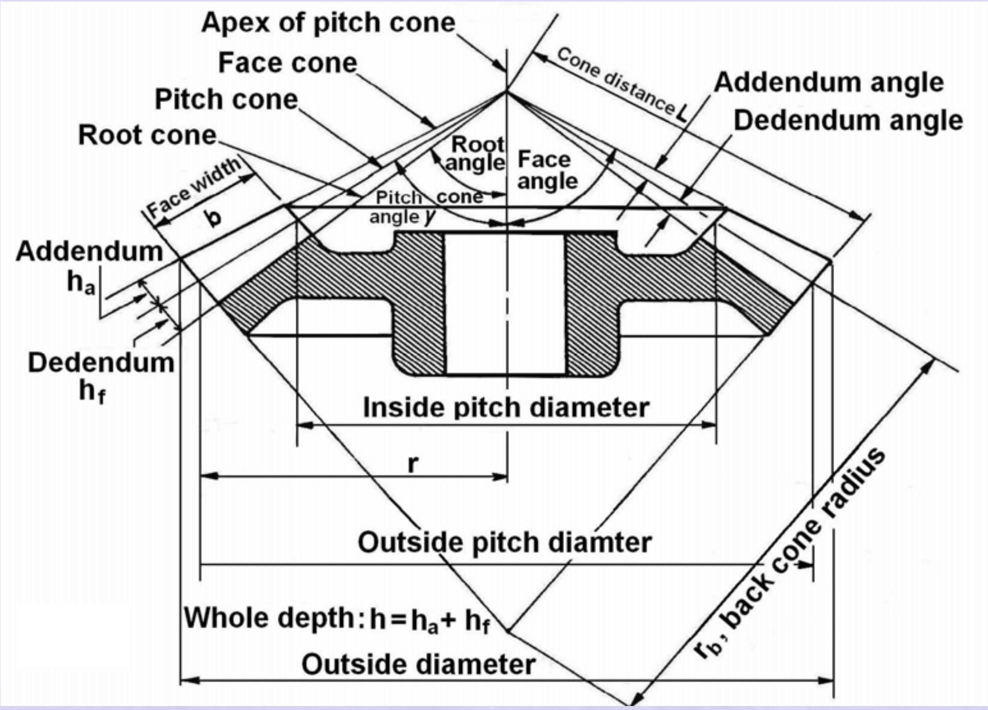



**Fig. 13.3 Bevel gear nomenclature**

$$Z_{v1} = \frac{2\pi r_{b1}}{p} = \frac{Z_1}{\cos\gamma_1} \quad \textbf{(13.1)}$$

$$Z_{v2} = \frac{2\pi r_{b2}}{p} = \frac{Z_2}{\cos\gamma} \quad \textbf{(13.2)}$$

where $Z_v$ is called the virtual number of teeth, p is the circular pitch of both the imaginary spur gears and the bevel gears. $Z_1$ and $Z_2$ are the number of teeth on the pinion and gear, $\gamma_1$ and $\gamma_2$ are the pitch cone angles of pinion and gears. It is a practice to characterize the size and shape of bevel gear teeth as those of an imaginary spur gear appearing on the developed back cone corresponding to Tredgold's approximation.

a) Bevel gear teeth are inherently non - interchangeable.

b) The working depth of the teeth is usually 2m, the same as for standard spur and helical gears, but the bevel pinion is designed with the larger addendum ( 0.7 working depth).

c) This avoids interference and results in stronger pinion teeth. It also increases the contact ratio.

d) The gear addendum varies from 1m for a gear ratio of 1, to 0.54 m for ratios of 6.8 and greater.

The gear ratio can be determined from the number of teeth, the pitch diameters or the pitch cone angles as,

$$i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{Z_2}{Z_1} = \frac{d_2}{d_1} = \tan\gamma_2 = \cot\gamma_1 \qquad \textbf{(13.3)}$$

Accepted practice usually imposes two limits on the face width

$$b \le 10\,m \quad \text{and} \quad b \le \frac{L}{3} \qquad \textbf{(13.4)}$$

Where L is the cone distance. Smaller of the two is chosen for design.

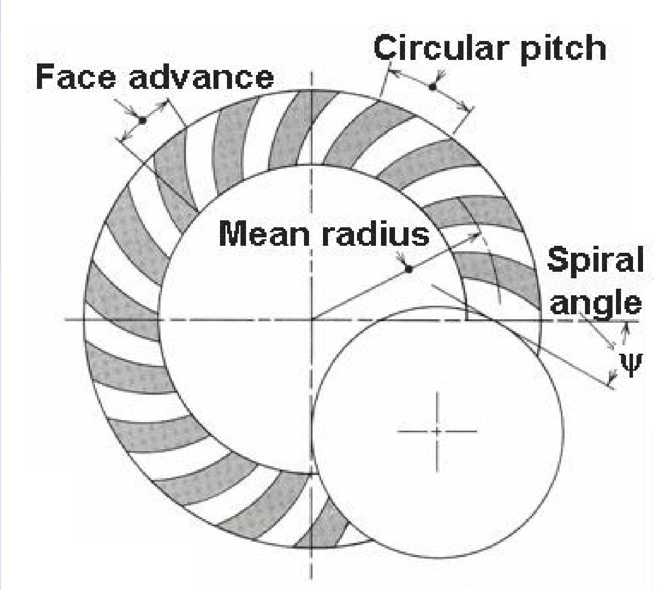



**Fig. 13.4 Illustration of spiral angle**

The Fig.13.4 illustrates the measurement of the spiral angle $\psi$ of a spiral bevel gear. Bevel gears most commonly have a pressure angle of $20^o$, and spiral bevels usually have a spiral angle $\psi$ of $35^o$.

**Fig.13.5 Zero bevel gears**

The Fig.13.5 illustrates Zero bevel gears, which are having curved teeth like spiral bevels. But they have a zero spiral angle.

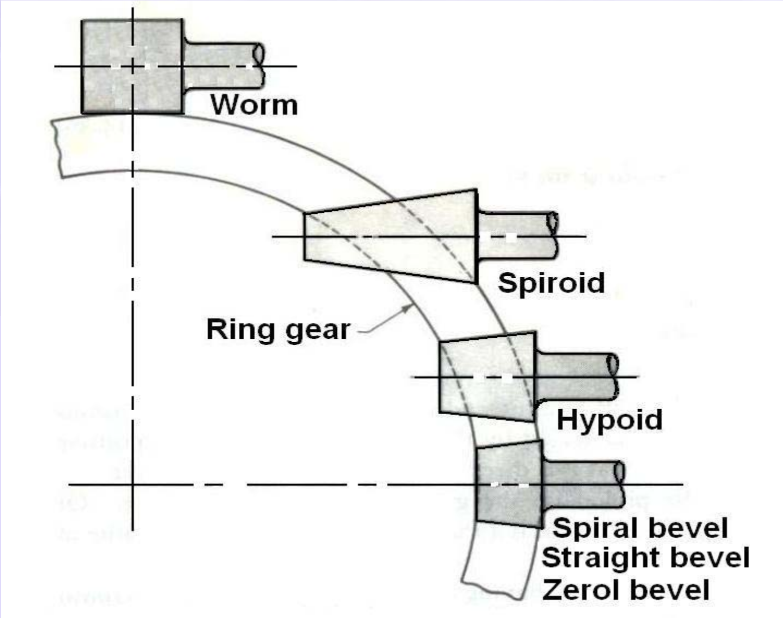



**Fig. 13.6 Comparison of intersecting and offset shaft bevel type gearings**

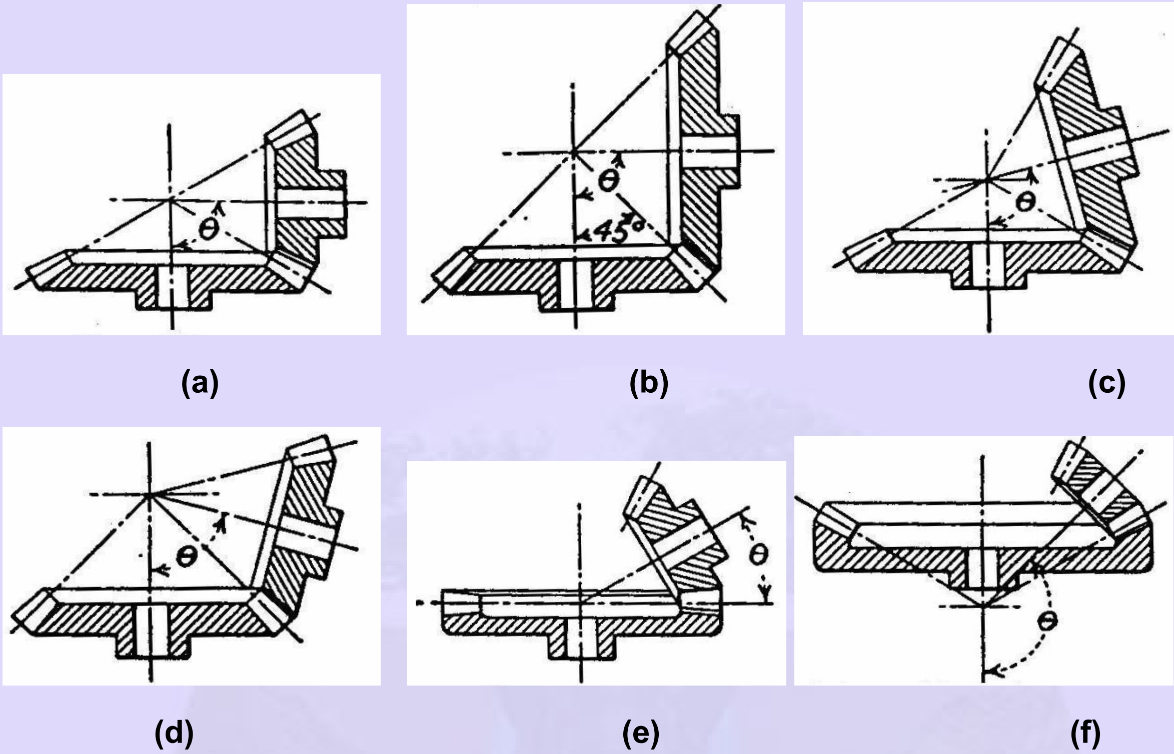



**Fig.13.7 Different types of bevel gears (a) Usual form, (b) Miter gears, (c), (d), (e) Crown gear, (f) Internal bevel gear**

## 13.3 FORCE ANALYSIS

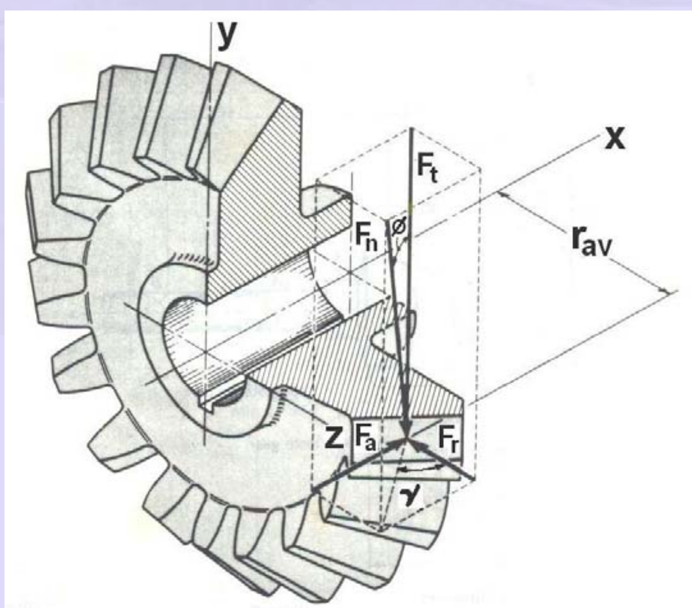



**Fig. 13.8 Gear tooth forces**

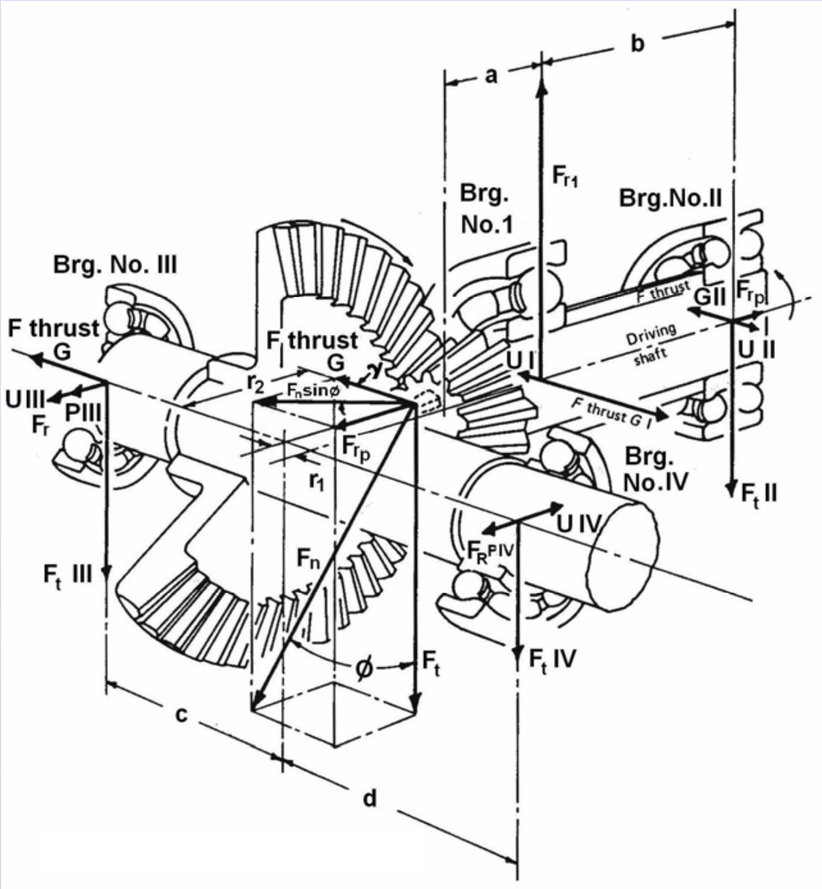



Fig. 13.9 Gear and shaft forces

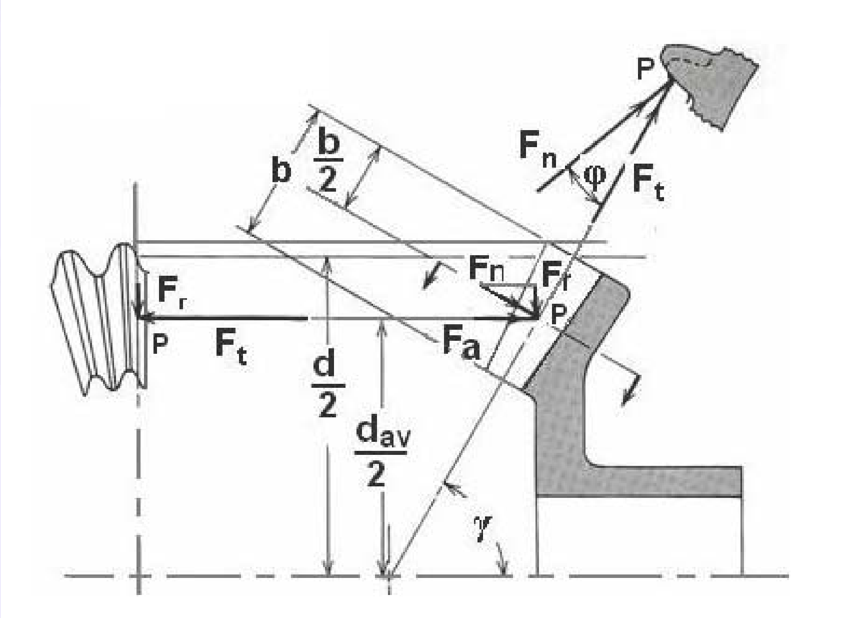



Fig. 13.10 Bevel gear - Force analysis

In Fig. 13.10, $F_n$ is normal to the pitch cone and the resolution of resultant tooth force $F_n$ into its tangential (torque producing), radial (separating) and axial (thrust) components is designated $F_t$, $F_r$ and $F_a$ respectively. An auxiliary view is needed to show the true length of the vector representing resultant force $F_n$ (which is normal to the tooth profile).

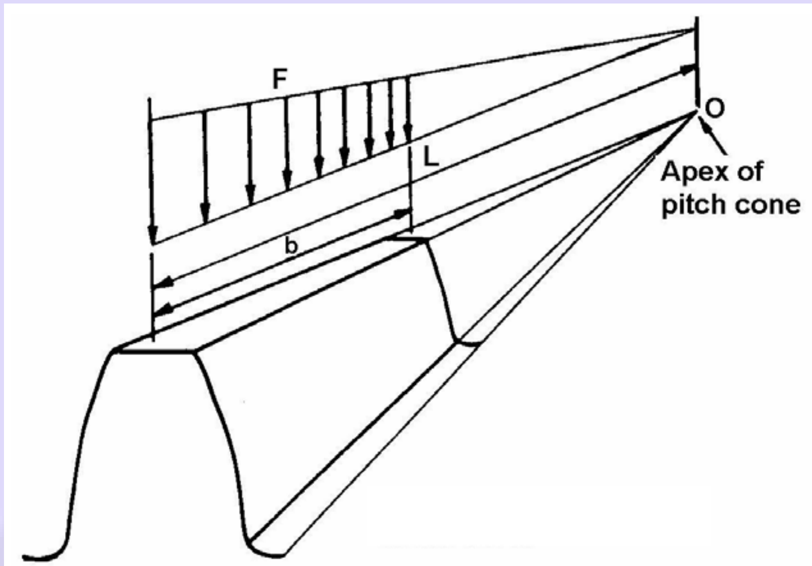



**Fig. 13.11 Linear tooth force distribution**

Resultant force $F_n$ is shown applied to tooth at the pitch cone surface and midway along tooth width b. It is also assumed that load is uniformly distributed along the tooth width despite the fact that the tooth width is larger at the outer end.

$$d_{av} = d - b \sin \quad (13.5)$$

$$V_{av} = \frac{\pi d_{av} n}{6000} \quad (13.6)$$

$$F_t = \frac{1000W}{v_{av}} \quad (13.7)$$

Where $V_{av}$ is in meters per second, $d_{av}$ is in meters, n is in revolutions per minute, $F_t$ is in N and W is power in kW.

$$F_n = F_t / \cos\varphi \quad (13.8)$$

$$F_r = F_n \cos\gamma = F_t \tan\varphi \cos \quad (13.9)$$

$$F_a = F_n \sin\gamma = F_t \tan\varphi \sin \quad (13.10)$$

For spiral bevel gear,

$$F_r = \frac{F_t}{\cos\psi}(\tan\varphi_n \cos\gamma \mp \sin\psi \sin\gamma) \qquad (13.11)$$

$$F_a = F_t(\tan\varphi_n \sin\gamma \pm \sin\psi \cos\gamma) \qquad (13.12)$$

Where $\mp$ or $\pm$ is used in the preceding equation, the upper sign applies to a driving pinion with right-hand spiral rotating clockwise as viewed from its large end and to a driving pinion with left-hand spiral rotating counter clock-wise when viewed from its large end. The lower sign applies to a left-hand driving pinion rotating clockwise and to a driving pinion rotating counter clockwise. Similar to helical gears, $\varphi_n$ is the pressure angle normal measured in a plane normal to the tooth.

## 13.4 TOOTH BENDING STRESS

The equation for bevel gear bending stress is the same as for spur gears as shown below:

$$\sigma_b = \frac{F_t}{bmJ} K_v K_o K_m \qquad (13.13)$$

Where, $F_t$ =Tangential load in N

m = module at the large end of the tooth in mm

b = Face width in mm

J = Geometry form factor based on virtual number of teeth from Fig. 13.12 and 13.13.

$K_v$ = Velocity factor, from Fig.13.14.

$K_o$ = Overload factor, Table 13.1.

$K_m$= Mounting factor, depending on whether gears are straddle mounted (between two bearings) or overhung (outboard of both bearings), and on the degree of mounting rigidity as shown in Table 13.2.

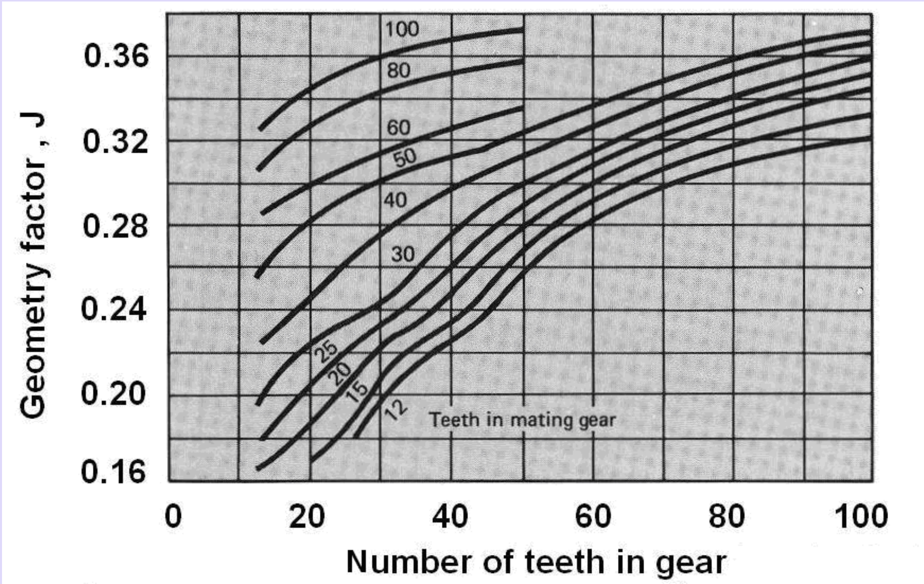



**Fig. 13.12 Number of teeth in gear for which geometry factor J is desired, pressure angle 20$^o$ and shaft angle 90$^o$**

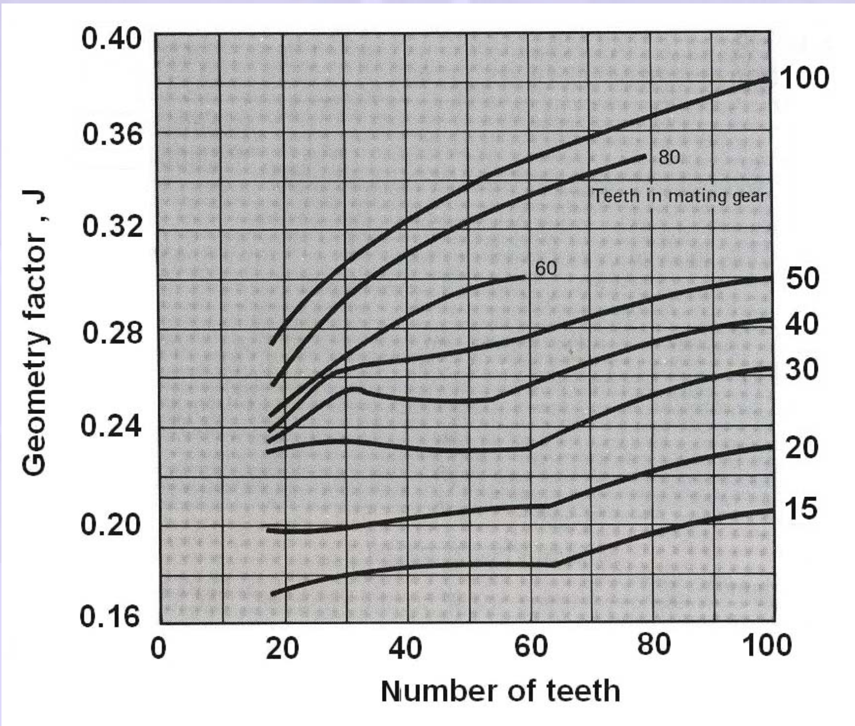



**Fig.13.13 Number of teeth in gear for which geometry factor J is desired, pressure angle 20$^o$, spiral angle 35$^o$ and shaft angle 90$^o$**

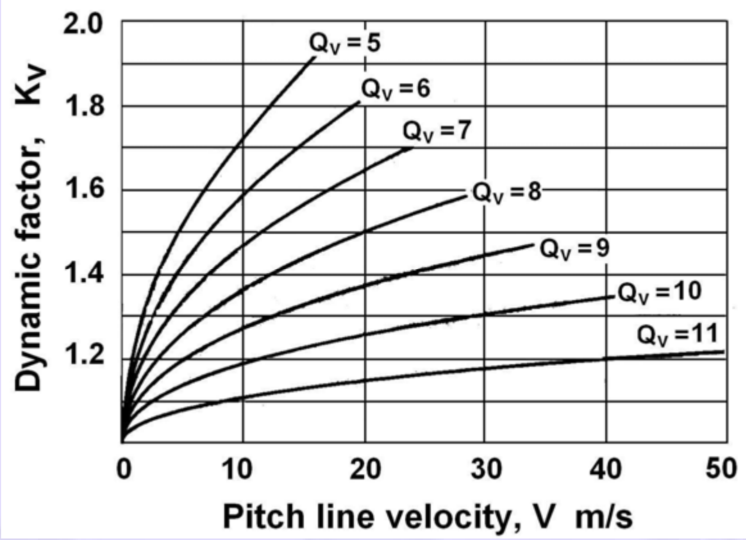
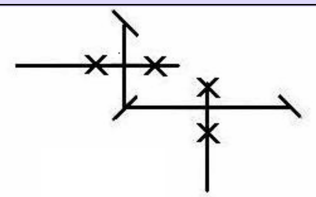
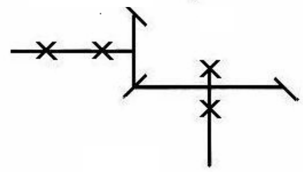
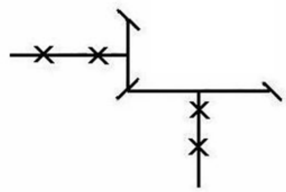



**Fig.13.14 Dynamic load factor, $K_v$**

**Table 13.1 -Overload factor $K_o$**

| | Driven Machinery | | |
|---|---|---|---|
| Source of power | Uniform | Moderate Shock | Heavy Shock |
| Uniform | 1.00 | 1.25 | 1.75 |
| Light shock | 1.25 | 1.50 | 2.00 |
| Medium shock | 1.50 | 1.75 | 2.25 |

**Table 13.2 Mounting factor $K_m$ for bevel gears**

| Mounting type | Mounting rigidity<br>Maximum to questionable | |
|---|---|---|
| Both gears are straddle-mounted |  | 1.0 to 1.25 |
| One gear straddle-mounted; the other overhung |  | 1.1 to 1.4 |
| Both gear overhung |  | 1.25 to 1.5 |

## 13.6 PERMISSIBLE TOOTH BENDING STRESS (AGMA)

Fatigue strength of the material is given by:

$$\boldsymbol{\sigma_e = \sigma_e' \, k_L \, k_v \, k_s \, k_r \, k_T \, k_f \, k_m} \qquad \textbf{(13.14)}$$

Where, $\sigma_e$' endurance limit of rotating-beam specimen

$k_L$ = load factor, = 1.0 for bending loads

$k_v$ = size factor, = 1.0 for m < 5 mm and

= 0.85 for m > 5 mm

$k_s$ = surface factor, taken from Fig.13.15 based on the ultimate strength of the material and for cut, shaved, and ground gears.

$k_r$ = reliability factor given in Table 13.3.

$k_T$ = temperature factor, = 1 for T≤ 120$^o$C and more than 120$^o$C, $k_T$ < 1 to be taken from AGMA standards

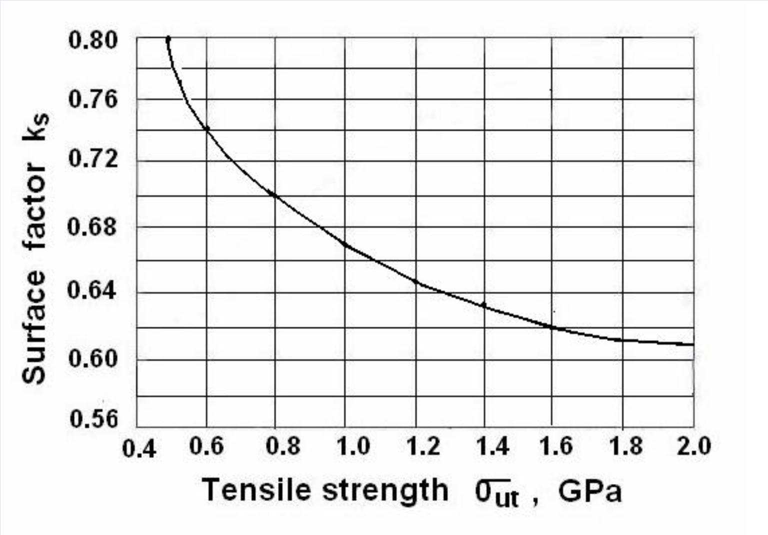



**Fig.13.15 Surface factor, $K_s$**

**Table 13.3 Reliability factor $K_r$**

| **Reliability factor R** | 0.50 | 0.90 | 0.95 | 0.99 | 0.999 | 0.9999 |
|---|---|---|---|---|---|---|
| **Factor Kr** | 1.000 | 0.897 | 0.868 | 0.814 | 0.753 | 0.702 |

$k_f$ = fatigue stress concentration factor. Since this factor is included in J factor its value is 1.

$k_m$ = Factor for miscellaneous effects. For idler gears subjected to two way bending, = 1. For other gears subjected to one way bending, the value is taken fromFig.13.16. Use $k_m$ = 1.33 for $\sigma_{ut}$ less than 1.4GPa.

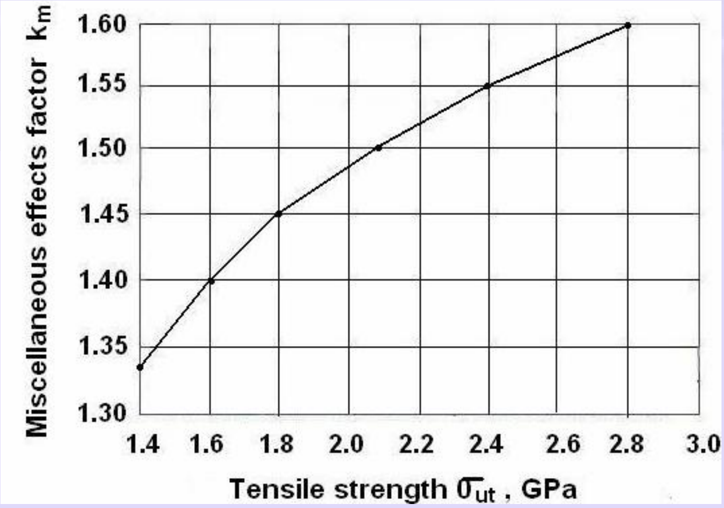



**Fig.13.16 Miscellaneous effects factor $K_m$**

Permissible bending stress is given by

$$[\sigma_b] = \frac{\sigma_e}{s} \qquad (13.15)$$

Hence the design equation from bending consideration is,

$$\boxed{\sigma_b \le [\sigma b]} \qquad (13.16)$$

Bevel gear surface fatigue stress can be calculated as for spur gears, with only two modifications.

$$\sigma_H = C_p \sqrt{\frac{F_t}{b\,d\,I} K_V K_o K_m} \qquad (13.17)$$

## 13.7 CONTACT STRESS:

1.23 times the $C_p$ values given in the Table13.4 are taken to account for a somewhat more localized contact area than spur gears.

**Table 13.4 Elastic Coefficient Cp for spur gears, in $(MPa)^{0.5}$**

| Pinion Material | Gear material | | | |
|---|---|---|---|---|
| (μ = 0.3 in all cases) | Steel | Cast iron | Al Bronze | Tin Bronze |
| Steel, E=307GPa | 191 | 166 | 162 | 158 |
| Cast iron, E = 131GPa | 166 | 149 | 149 | 145 |
| Al Bronze, E = 121GPa | 162 | 149 | 145 | 142 |
| Tin Bronze, E = 110GPa | 158 | 145 | 141 | 137 |

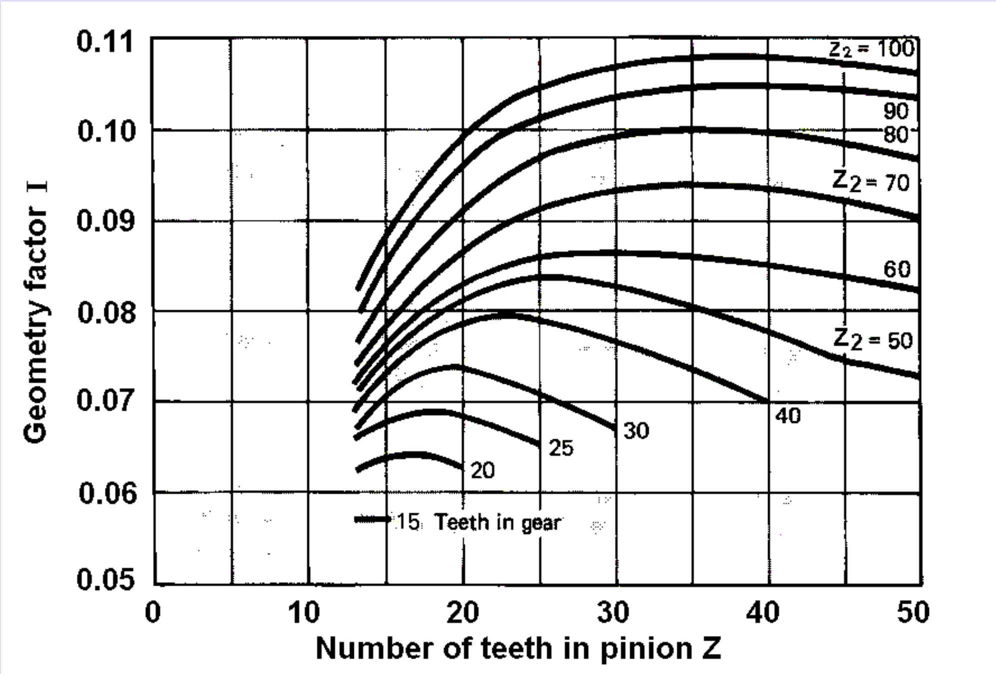



Fig.13.17 Geometry factor I for straight bevel gear pressure angle 20° and shaft angle 90°

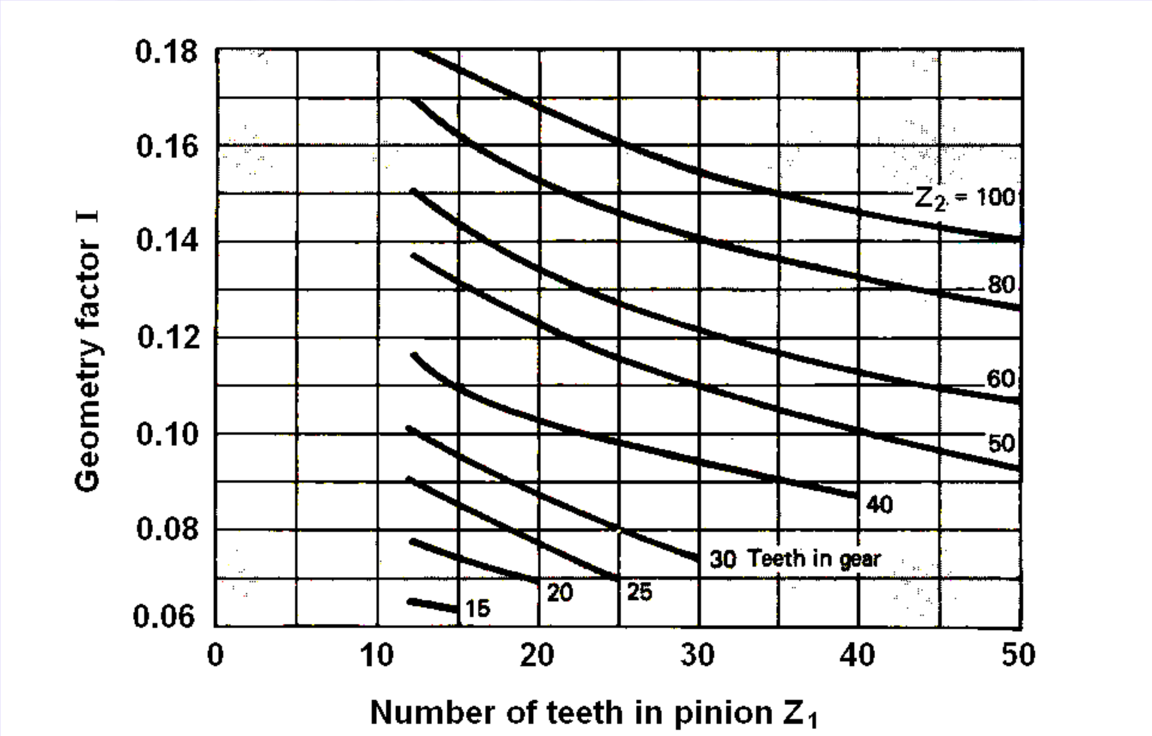



Fig.13.18 Geometry factor I for spiral bevel gear pressure angle 20°, spiral angle 35° and shaft angle 90°

Surface fatigue strength of the material is given by,

$$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T \qquad (13.18)$$

Where $\sigma_{sf}'$ = surface fatigue strength of the material given in Table 13.7

$K_L$ = Life factor given in Fig.13.19

**Table 13.7 Surface fatigue strength $\sigma_{sf}$ (MPa) for metallic spur gear, ($10^7$ cycle life 99% reliability and temperature < $120^o$ C)**

| **Material** | **σsf (MPa)** |
|---|---|
| Steel | 2.8 (Bhn) – 69 MPa |
| Nodular iron | 0.95 [ 2.8 (Bhn) – 69 MPa] |
| Cast iron, grade 20 | 379 |
| Cast iron, grade 30 | 482 |
| Cast iron, grade 40 | 551 |
| Tin Bronze, AGMA 2C ( 11% Sn) | 207 |
| Aluminium Bronze (ASTM b 148 – 52) (Alloy 9C – H.T ) | 448 |

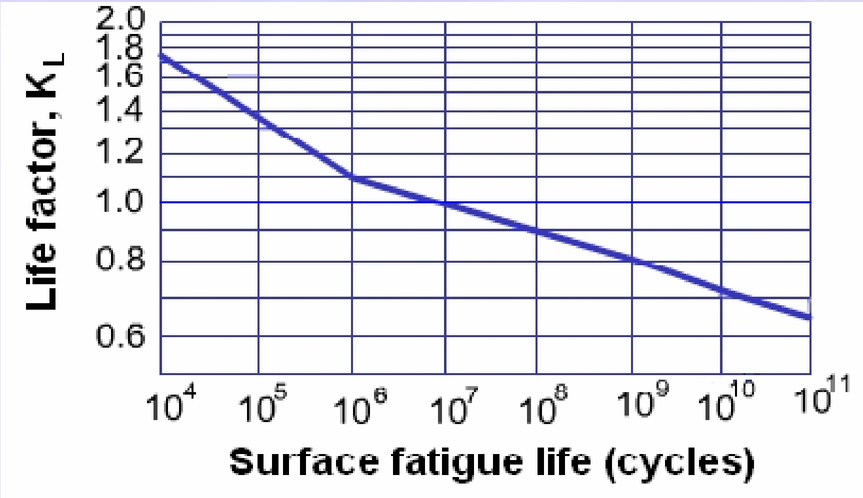



**Fig.13.19 Life factor $K_L$**

$K_H$ is hardness ratio factor, K the Brinell hardness of the pinion by Brinell hardness of the gear as given in Fig. 13.20.

$K_H$ = 1.0 for K < 1.2

$K_R$ = Reliability factor, given in Table 13.3.

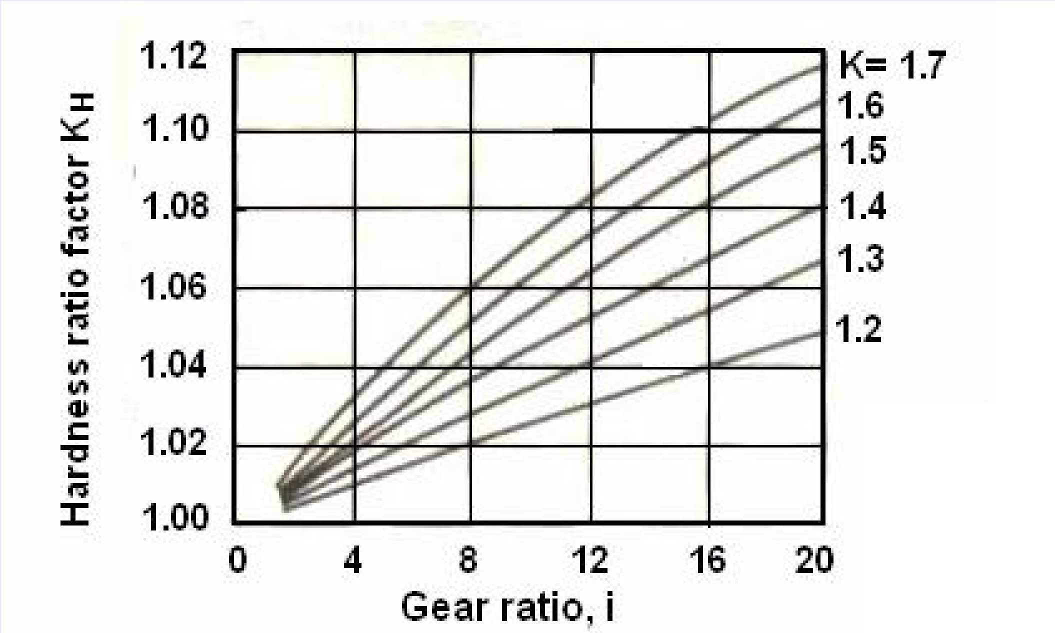



**Fig.13.20 Hardness ratio factor, $K_H$**

$K_T$ = temperature factor,

= 1 for T≤ 120°C based on lubricant temperature.

Above 120°C, it is less than 1 to be taken from AGMA standards.

Allowable surface fatigue stress for design is given by

$[\sigma_H] = \sigma_{Sf} / s$ (13.19)

Factor of safety s = 1.1 to 1.5

Hence Design equation is:

$\boxed{\sigma_H \leq [\sigma_H]}$ (13.20)

-------------------