

Lecture 4

GOVERNING PRINCIPLES AND LAWS

Learning Objectives

Upon completion of this chapter, the student should be able to:

- State Pascal's law.
- Write force power and force displacement relations.
- State practical applications of Pascal's law and evaluate the parameters.
- Explain the working of pressure booster and evaluate the parameters.
- Explain law of conservation of energy.
- Derive continuity and Bernoulli's equation.
- Modify Bernoulli's equation to energy equation.
- State Torricelli's theorem and workout related problems.
- State siphon principle and workout problems.

1.1 Introduction

Fluid power systems are designed using all the principles learned in fluid mechanics. It is appropriate to briefly review these principles before proceeding with our study of the applications. One of the underlying postulates of fluid mechanics is that, for a particular position within a fluid at rest, the pressure is the same in all directions. This follows directly from Pascal's Law. A second postulate states that fluids can support shear forces only when in motion. These two postulates define the characteristics of fluid media used to transmit power and control motion. This chapter deals with fundamental laws and equations which govern the fluid flow which is essential for the rational design of fluid power components and systems. Traditional concepts such as continuity, Bernoulli's equation and Torricelli's theorem are presented after a brief review on mechanics.

1.2 Brief review of Mechanics

Fluid power deals with conversion of hydraulic power to mechanical power. Therefore it is essential to understand the concept of energy and power.

1.2.1 Energy

Energy is defined as the ability to perform work. If a force acts on a body and moves the body through a specified distance in the direction of its application, a work has been done on the body. The amount of this work equals the product of the force and distance where both the force and distance are measured in the same direction. Mathematically we can write

$$W_D = Fd$$

where F is the force (N), d is the distance (m) and W_D is the work done (J or Nm). In the SI system, a joule (J) is the work done when a force of 1 N acts through a distance of 1 m. Since work equals force times distance, we have

$$1 \text{ J} = 1 \text{ N} \times 1 \text{ m} = 1 \text{ Nm}$$

Thus, we have

$$\text{Energy (J)} = F \text{ (N)} \times d \text{ (m)}$$

The transfer of energy is an important consideration in the operation of fluid power systems. Energy from a prime mover is transferred to a pump via a rotating motor shaft and couplings. The pump converts this mechanical energy into hydraulic energy by increasing the fluid pressure. The pressurized fluid does work on hydraulic actuators. An actuator converts the hydraulic energy into mechanical energy and moves the external load. Not all the input mechanical energy is converted into useful work. There are frictional losses through valves, fittings and other system control components.

These losses show up as heat loss that is lost to the atmosphere with the increase in the fluid temperature.

1.2.2 Power

It is defined as the rate of doing work. Thus, the power input to the hydraulic system is the rate at which an actuator delivers energy to the external load. Similarly, the rate at which an actuator delivers energy to the external load is equal to the power output of a hydraulic system. The power output is determined by the requirements of the external load.

A hydraulic system is used because of its versatility in transferring power. The versatility includes the advantages of variable speed, reversibility, overload protection, high power-to-weight ratio and immunity to damage under a stalled condition:

$$\text{Power} = P = \frac{Fd}{t}$$

or

$$P = Fv$$

where F is the force (N), v is the velocity (m/s) and P is the power (N m/s or W). In the SI system, 1 watt (W) of power is the rate at which 1 J of work is done per second:

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

In SI units we have

$$1 \text{ W} = \frac{1 \text{ Joule}}{\text{s}} = 1 \text{ N m / s}$$

Thus, we have

$$\text{Power (W)} = \frac{\text{Work (N m)}}{\text{Time(s)}}$$

Balancing the units, we can write

$$\begin{aligned} \text{Hydraulic power (W)} &= \text{Pressure} \times \text{Flow} \\ &= p \text{ (N / m}^2\text{)} \times Q \text{ (m}^3\text{/s)} \\ &= p \times Q \text{ (N m / s)} = p \times Q \text{ (W)} \end{aligned}$$

It is usual to express flow rate in liters/minute (LPM) and pressure in bars. To calculate hydraulic power using these units, a conversion has to be made. Thus,

$$\begin{aligned} Q \text{ (L/min)} &= Q/60 \text{ (L/s)} \\ \Rightarrow Q \left(\frac{\text{L}}{\text{min}} \right) &= \frac{Q}{60} \left(\frac{\text{L}}{\text{s}} \right) = \frac{Q}{60 \times 10^3} \left(\frac{\text{m}^3}{\text{s}} \right) \\ \Rightarrow p \text{ (bar)} &= p \times 10^5 \frac{\text{N}}{\text{m}^2} \end{aligned}$$

Hydraulic power is

$$\begin{aligned} Q \left(\frac{1}{\text{min}} \right) \times p \text{ (bar)} \times \frac{1 \times 10^5}{60 \times 10^3} \left(\frac{\text{m}^3}{\text{s}} \times \frac{\text{N}}{\text{m}^2} \right) \\ \Rightarrow Q \times p \text{ (bar)} \times \frac{1 \times 10^3}{600} \text{ (W)} = \frac{Q \text{ (LPM)} \times p \text{ (bar)}}{600} \text{ (kW)} \end{aligned}$$

Thus, hydraulic power (kW) is

$$\frac{\text{Flow (LPM)} \times \text{Pressure (bar)}}{600}$$

In the SI metric system, all forms of power are expressed in watt. The pump head H_p in units of meters can be related to pump power in units of watt by using $P = \gamma h$. So

$$H_p = \frac{\text{Pump hydraulic power (W)}}{\gamma \left(\frac{\text{N}}{\text{m}^3} \right) \times Q \left(\frac{\text{m}^3}{\text{s}} \right)}$$

The above equation can also be used to find a motor head where H_p is replaced by H_m . The hydraulic power is replaced by the motor hydraulic power and Q represents the motor flow rate.

The mechanical power output (brake power or torque power) delivered by a hydraulic motor can be found by the following equation

$$\text{Power (kW)} = \frac{T (\text{N m}) \times \omega (\text{rad/s})}{1000} = \frac{T (\text{N m}) \times N (\text{rpm})}{9550}$$

Where T is the torque and ω or N is the angular speed.

1.3 Pascal's Law

Pascal's law states that the pressure exerted on a confined fluid is transmitted undiminished in all directions and acts with equal force on equal areas and at right angles to the containing surfaces. In Fig. 1.1, a force is being applied to a piston, which in turn exerts a pressure on the confined fluid. The pressure is equal everywhere and acts at right angles to the containing surfaces. Pressure is defined as the force acting per unit area and is expressed as

$$\text{Pressure} = \frac{F}{A}$$

where F is the force acting on the piston, A is the area of the piston and p is the pressure on the fluid.

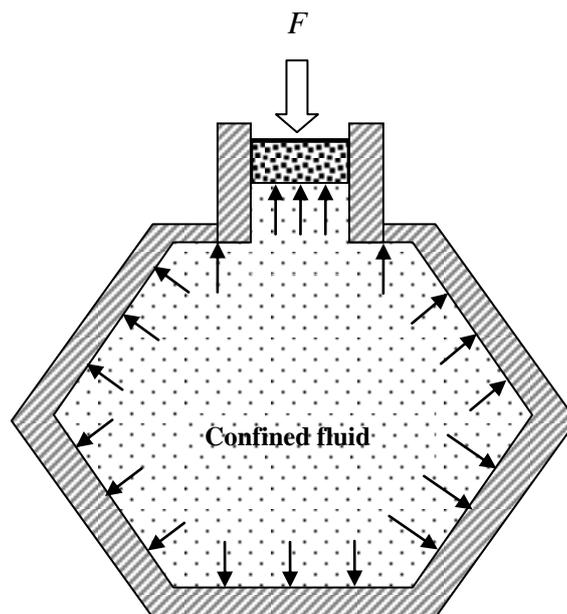


Figure 1.1 Illustration of Pascal's law

1.3.1 Multiplication of Force

The most useful feature of fluid power is the ease with which it is able to multiply force. This is accomplished by using an output piston that is larger than the input piston. Such a system is shown in Fig. 1.2.

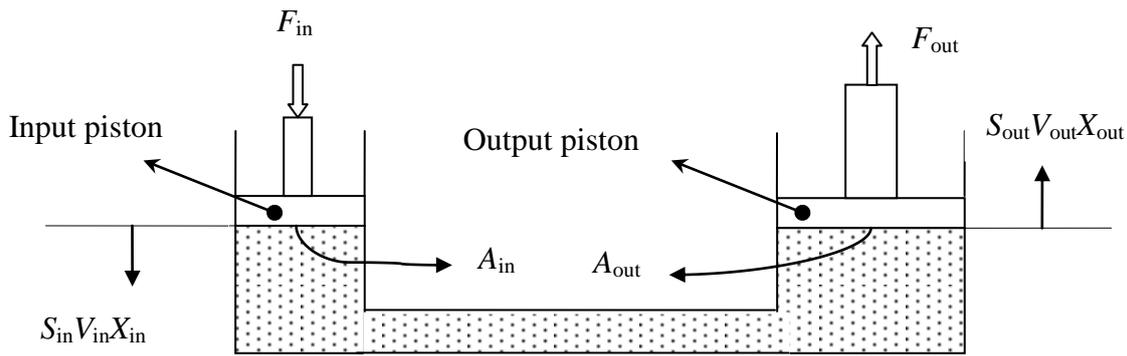


Figure 1.2 Multiplication of force

This system consists of an input cylinder on the left and an output cylinder on the right that is filled with oil. When the input force is F_{in} on the input piston, the pressure in the system is given by

$$P = \frac{F_{out}}{A_{out}}$$

$$\Rightarrow F_{out} = PA_{out} = \frac{F_{in}}{A_{in}} A_{out} = \frac{A_{out}}{A_{in}} F_{in}$$

Here to obtain the output force, the input force is multiplied by a factor that is equal to the ratio of the output piston area to the input piston area. If the output piston area is x times the input piston area, then the output force is x times the input force. Generally, the cross-sectional area of the piston is circular. The area is given by

$$A = \pi d^2 / 4$$

Hence, the above equation can be written as

$$F_{out} = \frac{d_{out}^2}{d_{in}^2} F_{in}$$

$$\Rightarrow \frac{F_{out}}{F_{in}} = \frac{d_{out}^2}{d_{in}^2}$$

The conservation of energy is very fundamental principle. It states that energy can neither be created nor destroyed. At first sight, multiplication of force as depicted in Fig.1.2 may give the impression that something small is turned into something big. But this is wrong, since the large piston on the right is only moved by the fluid displaced by the small piston on left. Therefore, what has been gained in force must be sacrificed in piston travel displacement. Now we shall mathematically derive force displacement relation and force power relation.

- 1. Force displacement relation:** A hydraulic oil is assumed to be incompressible; hence, the volume displaced by the piston is equal to the volume displaced at the output piston:

$$V_{in} = V_{out}$$

Since the volume of a cylinder equals the product of its cross-sectional area and its height, we have

$$A_{in} S_{in} = A_{out} S_{out}$$

where S_{in} is the downward displacement of the input piston and S_{out} is the upward displacement of the output piston:

$$\frac{S_{in}}{S_{out}} = \frac{A_{out}}{A_{in}}$$

Comparing

$$\frac{F_{out}}{F_{in}} = \frac{A_{out}}{A_{in}} = \frac{S_{in}}{S_{out}} \quad (1.1)$$

2. **Force power relation:** A hydraulic oil is assumed to be incompressible; hence, the quantity of oil displaced by the input piston is equal to the quantity of oil gained and displaced at the output piston:

Flow rate is the product of area and volume of fluid displaced in a specified time

$$\begin{aligned} Q_{in} &= Q_{out} \\ \Rightarrow A_{in} V_{in} &= A_{out} V_{out} \\ \Rightarrow \frac{A_{out}}{A_{in}} &= \frac{V_{in}}{V_{out}} \end{aligned} \quad (1.2)$$

Comparing Equations. (1.1) and (1.2) we get

$$\frac{A_{out}}{A_{in}} = \frac{V_{in}}{V_{out}} = \frac{F_{out}}{F_{in}} = \frac{S_{in}}{S_{out}}$$

From the above equation, we get

$$F_{in} S_{in} = F_{out} S_{out}$$

(Work done)_{in} = (Work done)_{out}

or

We know that

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$\Rightarrow F_{in} v_{in} = F_{out} v_{out}$$

or

$$(\text{Power})_{in} = (\text{Power})_{out}$$

Example 1.1

An input cylinder with a diameter of 30 mm is connected to an output cylinder with a diameter of 80 mm (Fig. 1.3). A force of 1000 N is applied to the input cylinder.

(a) What is the output force?

(b) How far do we need to move the input cylinder to move the output cylinder 100 mm?

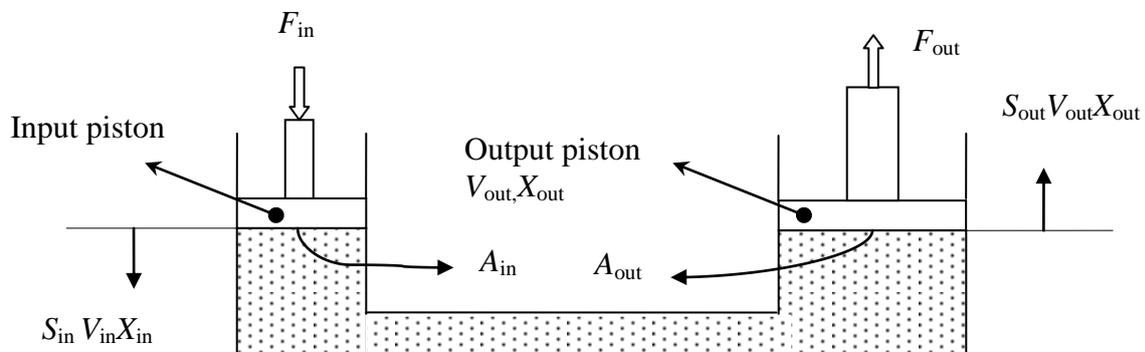


Figure 1.3

Solution: Since the volume of a cylinder equals the product of its cross-sectional area and its height, we have

$$A_{in} X_{in} = A_{out} X_{out}$$

where X_{in} is the downward movement of the input piston and X_{out} is the upward movement of the output piston. Hence we get

$$\frac{X_{out}}{X_{in}} = \frac{A_{in}}{A_{out}}$$

The piston stroke ratio X_{out} / X_{in} equals the piston area ratio A_{in} / A_{out} . For a piston area of 10, the output force F_{out} increases by a factor of 10, but the output motion decreases by a factor of 10.

Thus, the output force is greater than the input force, but the output movement is less than the input force and the output movement is less than the input movement. Hence, we can write by combining equations

$$A_{in} X_{in} = A_{out} X_{out} \text{ and } \frac{X_{out}}{X_{in}} = \frac{A_{in}}{A_{out}}$$

that

$$\begin{aligned} \frac{F_{out}}{F_{in}} &= \frac{X_{in}}{X_{out}} \\ \Rightarrow W_{in} &= W_{out} \end{aligned}$$

Hence, the input work equals the output work.

Given $F_{in} = 1000 \text{ N}$, $A_1 = 0.7854 \times 30^2 \text{ mm}^2$ and $A_2 = 0.7854 \times 80^2 \text{ mm}^2$, $S_{out} = 1000 \text{ mm}$.

To calculate S_{in} and F_2 .

(a) **Force on the large piston F_2 :** By Pascal's law, we have

$$\begin{aligned} \frac{F_1}{F_2} &= \frac{A_1}{A_2} \\ \Rightarrow F_2 &= \frac{A_2 \times F_1}{A_1} = \frac{1000 \text{ N}}{0.7854 \times 30^2} \times 0.7854 \times 80^2 \\ \Rightarrow F_2 &= 7111.1 \text{ N} \end{aligned}$$

(b) **Distance moved by the large piston S_{out} :** We also know by the conservation of energy that

$$\begin{aligned} \frac{F_1}{F_2} &= \frac{S_{out}}{S_{in}} \\ \Rightarrow S_{in} &= \frac{S_{out} \times F_2}{F_1} = \frac{1000 \times 7111.1}{1000} \\ \Rightarrow S_{in} &= 7111.11 \text{ mm} \end{aligned}$$

Example 1.2

A force of $P = 850 \text{ N}$ is applied to the smaller cylinder of a hydraulic jack (Fig. 1.4). The area a of the small piston is 15 cm^2 and the area A of the larger piston is 150 cm^2 . What load W can be lifted on the larger piston (a) if the pistons are at the same level, (b) if the large piston is 0.75 m below the smaller one? The mass density ρ of the liquid in the jack is 103 kg/m^3 .

Solution: A diagram of a hydraulic jack is shown in Fig. 1.4. A force F is applied to the piston of the small cylinder which forces oil or water into the large cylinder thus raising the piston supporting the load W . The force F acting on the area a produces a pressure p_1 that is transmitted equally in all directions through the liquid. If the pistons are at the same level, the pressure p_2 acting on the larger piston must equal p_1 as per Pascal's law

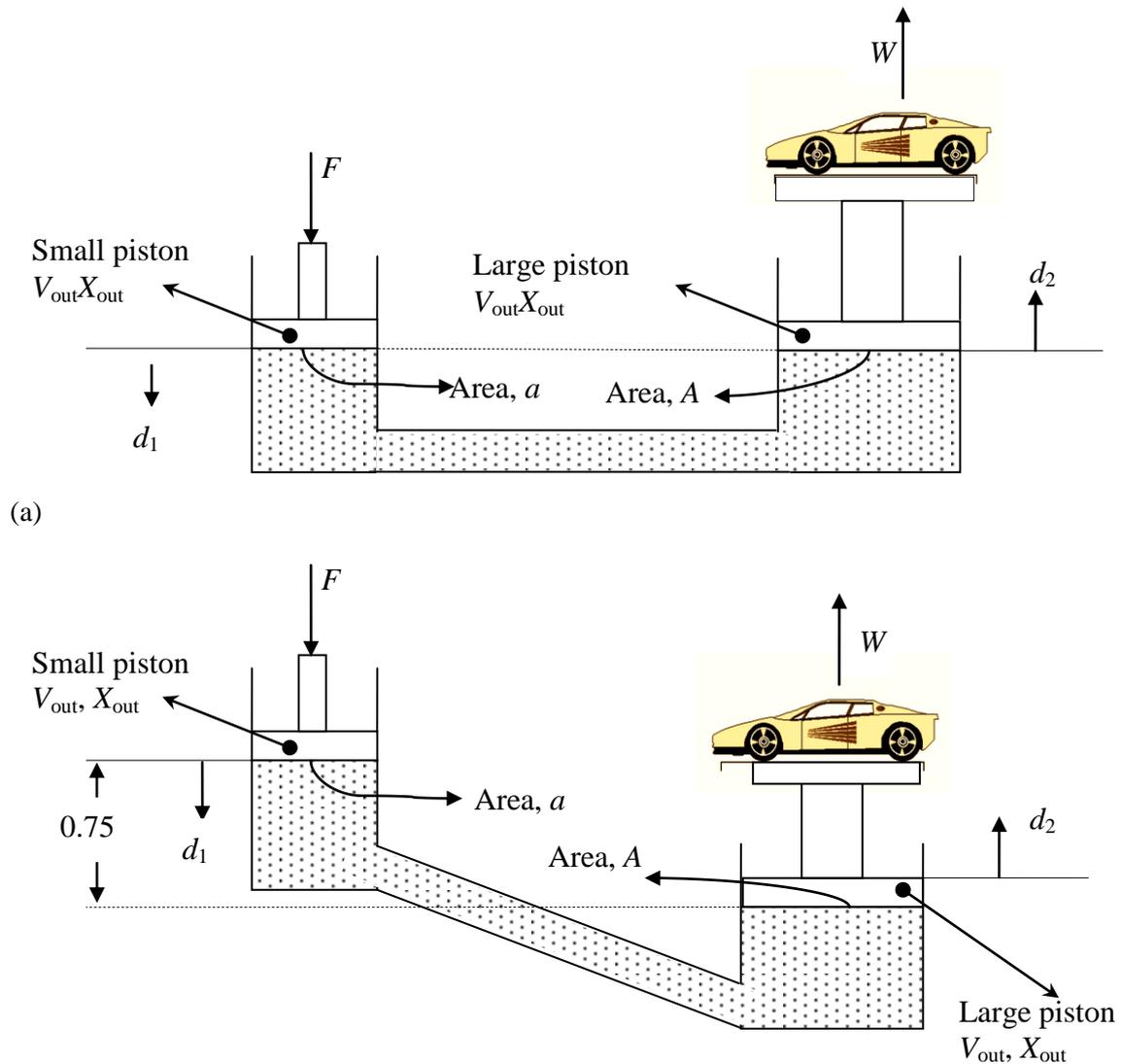


Figure 1.4 (a) Pistons are at same level. (b) Pistons are at different level.

We know that

$$p_1 = \frac{F}{a} \text{ and } p_2 = \frac{W}{A}$$

If $p_1 = p_2$, a small force can raise a larger load W . The jack has a mechanical advantage of A/a .

(a) Now $P=850\text{ N}$, $a=15/1000\text{ m}^2$, $A=150/10000\text{ m}^2$. Using Pascal's law we can write

$$\begin{aligned} p_1 &= p_2 \\ \Rightarrow \frac{F}{a} &= \frac{W}{A} \\ \Rightarrow W &= \frac{F \times A}{a} = \frac{850 \times 1.5}{0.15} = 8500\text{ N} \end{aligned}$$

Now

$$\text{Mass lifted} = \frac{W}{g} = \frac{8500}{9.81} = 868\text{ kg}$$

(b) If the larger piston is a distance h below the smaller, the pressure p_2 is greater than p_1 , due to the head h , by an amount ρg where ρ is the mass density of the liquid:

$$p_2 = p_1 + \rho gh$$

Now

$$p_1 = \frac{F}{a} = \frac{850}{15 \times 10^{-4}} = 56.7 \times 10^4 \text{ N/m}^2$$

$$\rho = 103 \text{ kg/m}^3$$

$$h = 0.75 \text{ m}$$

So

$$p_2 = 56.7 \times 10^4 + (103 \times 9.81) \times 0.75 \\ = 57.44 \times 10^4 \text{ N/m}^2$$

Now

$$W = p_2 A = 57.44 \times 10^4 \times 150 \times 10^{-4} = 8650 \text{ N}$$

Therefore

$$\text{Mass lifted} = \frac{W}{g} = 883 \text{ kg}$$

Example 1.3

Two hydraulic cylinders are connected at their piston ends (cap ends rather than rod ends) by a single pipe (Fig. 1.5). Cylinder A has a diameter of 50 mm and cylinder B has a diameter of 100 mm. A retraction force of 2222 N is applied to the piston rod of cylinder A. Determine the following:

- Pressure at cylinder A.
- Pressure at cylinder B.
- Pressure in the connection pipe.
- Output force of cylinder B.

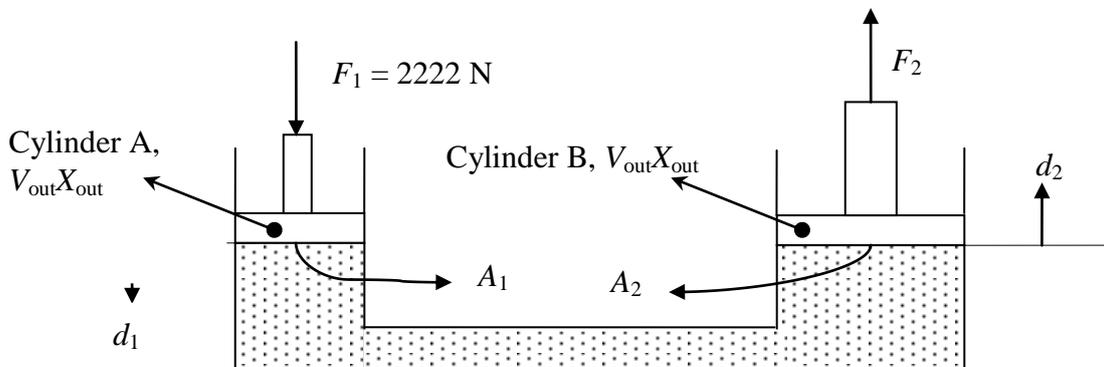


Figure 1.5

Area of the piston of cylinder A is

$$\frac{\pi}{4} (50)^2 = 1963.5 \text{ mm}^2$$

Area of the piston of cylinder B is

$$\frac{\pi}{4} (100)^2 = 7853.8 \text{ mm}^2$$

- Pressure in cylinder A is given by

$$\frac{\text{Force}}{\text{Area}} = \frac{2222}{1963.5} = 1.132 \frac{\text{N}}{\text{mm}^2} = 1.132 \text{MPa}$$

(b) By Pascal's law, pressure in cylinder A = pressure in cylinder B = 1.132MPa

(c) By Pascal's law, pressure in cylinder A = pressure in cylinder B = pressure in the pipe line = 1.132 MPa

(d) Force on the large piston (cylinder B) F_2 : By Pascal's law, we have

$$\begin{aligned} \frac{F_1}{F_2} &= \frac{A_1}{A_2} \\ \Rightarrow F_2 &= \frac{A_2 \times F_1}{A_1} = \frac{2222 \text{N}}{1963.5 \text{mm}^2} \times 7853.8 \text{mm}^2 = 8888 \text{N} \end{aligned}$$

Example 1.4

A pump delivers oil to a cylindrical storage tank, as shown in Fig. 1.6. A faulty pressure switch, which controls the electric motor driving the pump, allows the pump to fill the tank completely. This causes the pressure p_1 near the base of the tank to build to 103.4 kPa.

(a) What force is exerted on the top of the tank?

(b) What does the pressure difference between the tank top and point 1 say about Pascal's law?

(c) What must be true about the magnitude of system pressure if the changes in pressure due to elevation changes can be ignored in a fluid power system (assume the specific gravity of oil to be 0.9).

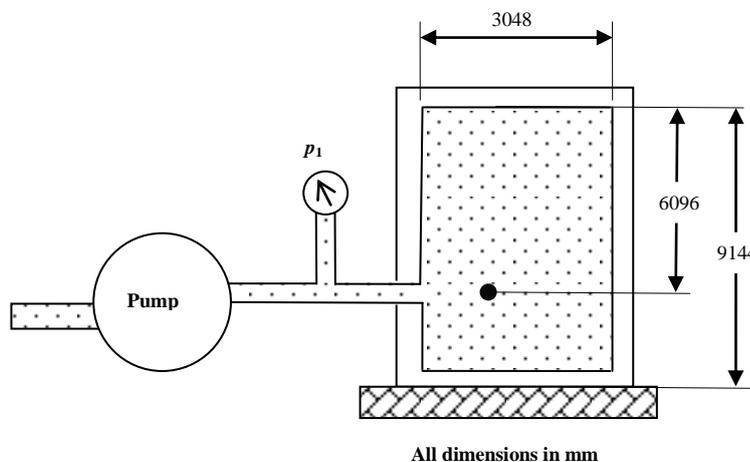


Figure 1.6

(a) We know that

$$\begin{aligned} \Delta p &= \gamma(\Delta H) \\ &= 900 \times 9.81 \frac{\text{N}}{\text{m}^3} \times (6.096 \text{ m}) \\ &= 53821.6 \text{ Pa} \\ &= 53.822 \text{ kPa} \end{aligned}$$

Thus,

$$F_{\text{top of tank}} = 103.4 - 53.82 = 49.58 \text{ kPa}$$

(b) Now

$$\begin{aligned}
 F &= \text{Pressure} \times \text{Area} \\
 &= 49.58 \times 1000 \times \frac{\pi}{4} (3.048)^2 \\
 &= 361755 \text{ N} \\
 &= 361.76 \text{ kN}
 \end{aligned}$$

Pascal's law states that pressure in a static body of fluid is transmitted equally only at the same elevation level. Pressure increases with depth and vice versa in accordance with the following equation: $\Delta p = \gamma(\Delta H)$.

(c) Changes in pressure due to elevation changes can be ignored in a fluid power system as long as they are small compared to the magnitude of the system pressure produced at the pump discharge port.

Example 1.5

The hydraulic jack, shown in Fig. 1.7, is filled with oil. The large and small pistons have diameters of 75 and 25 mm, respectively. What force on the handle is required to support a load of 8896 N? If the force moves down by 125 mm, how far is the weight lifted?

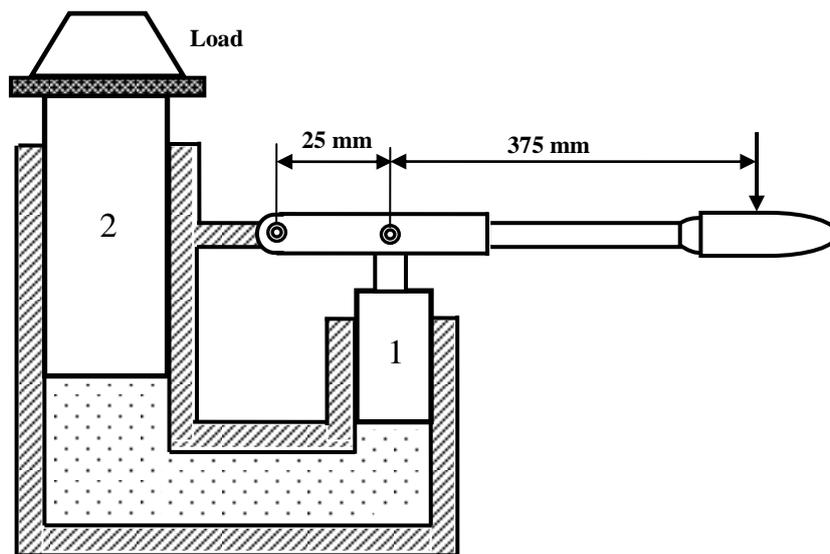


Figure 1.7

Solution: The relation for the lever force system gives

$$F \times 400 = F_1 \times 25$$

$$\Rightarrow F = \frac{F_1}{16}$$

Now since the oil pressure must remain the same everywhere, we have $p_1 = p_2$. Therefore

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow \frac{F_1}{\pi \times 0.025^2 / 4} = \frac{8896}{\pi \times 0.075^2 / 4}$$

$$\Rightarrow F_1 = 988.44 \text{ N}$$

From the relation obtained above, we get

$$F = \frac{F_1}{16} = \frac{988.44}{16} = 61.78 \text{ N}$$

The force moves by 125 mm. The force displacement diagram is shown in Fig. 1.8

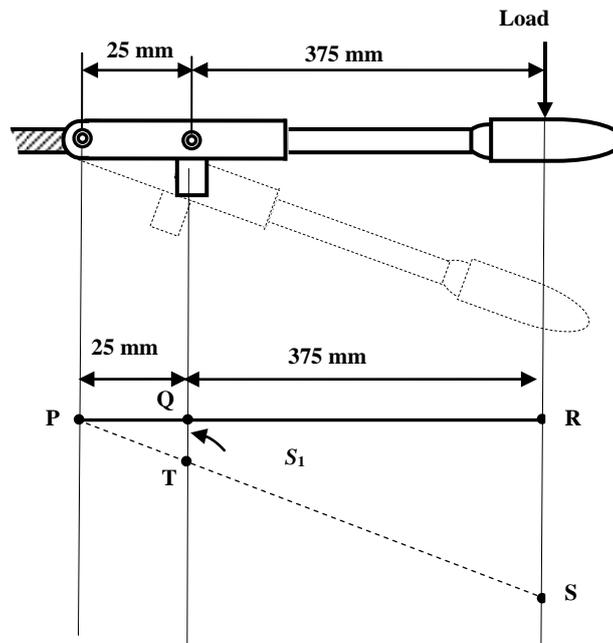


Figure 1.8 Force displacement diagram

From Fig. 1.8 we have

$$\begin{aligned} \frac{RS}{QT} &= \frac{RP}{PQ} \\ \Rightarrow \frac{RS}{S_1} &= \frac{RP}{PQ} \\ \Rightarrow S_1 &= \frac{RS \times PQ}{RP} = \frac{150 \times 25}{400} = 9.375 \text{ mm} \end{aligned}$$

Now

$$\begin{aligned} A_1 S_1 &= A_2 S_2 \\ \Rightarrow \pi \times \frac{25^2}{4} \times 9.375 &= \pi \times \frac{75^2}{4} \times S_2 \\ \Rightarrow S_2 &= \frac{1}{9} \times 9.375 \cong 1 \text{ mm} \end{aligned}$$

Hence, 150 mm stroke length of lever moves the load of 8896 N by only 1 mm. In other words, mechanical advantage is obtained at the expense of distance traveled by the load.

Example 1.6

In the hydraulic device shown in Fig.1.9, calculate the output torque T_2 , if the input torque $T_1 = 10$ N-cm. Use the following data: radius $R_1 = 2$ cm, diameter $d_1 = 8$ cm, radius $R_2 = 4$ cm, diameter $d_2 = 24$ cm.

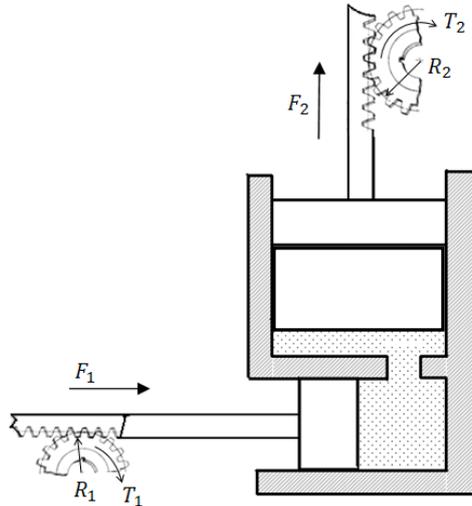


Figure 1.9

Solution: We can use Pascal's law and write

$$\begin{aligned}
 p_1 &= p_2 \\
 \Rightarrow \frac{F_1}{A_1} &= \frac{F_2}{A_2} \\
 \Rightarrow \frac{F_1}{d_1^2} &= \frac{F_2}{d_2^2}
 \end{aligned}$$

Since torque $T = F \cdot R$ which implies $F = T / R$, we can also write

$$\begin{aligned}
 \frac{T_1}{R_1 d_1^2} &= \frac{T_2}{R_2 d_2^2} \\
 \Rightarrow T_2 &= \frac{T_1 \times R_2 d_2^2}{R_1 d_1^2} \\
 \Rightarrow T_2 &= \frac{10 \times 4 \times 24^2}{2 \times 8^2} = 180 \text{ N cm}
 \end{aligned}$$

Alternate method: We know that

Torque $T = \text{Force} \times \text{Radius of the gear}$

Consider gear 1. We have

$$\begin{aligned}
 T_1 &= F_1 R_1 \\
 \Rightarrow F_1 &= \frac{T_1}{R_1} = \frac{10}{2} = 5 \text{ N}
 \end{aligned}$$

Now

$$p_1 = \frac{F_1}{A_1}$$

where A_1 is the area of the horizontal piston given by

$$\frac{\pi}{4}(d^2) = \frac{\pi}{4}(8^2) = 49.84 \text{ cm}^2$$

So

$$p_1 = \frac{5}{49.84} = 0.100 \text{ N/cm}^2$$

According to Pascal's law, $p_1 = p_2 = 0.100 \text{ N/cm}^2$. Here

$$p_2 = \frac{F_2}{A_2}$$

where A_2 is the area of the horizontal piston given by

$$\frac{\pi}{4}(d^2) = \frac{\pi}{4}(24^2) = 452.16 \text{ cm}^2$$

So

$$p_2 = \frac{F_2}{A_2}$$

$$\Rightarrow 0.100 = \frac{F_2}{452.16}$$

$$\Rightarrow F_2 = 45.216 \text{ N}$$

Now

$$T_2 = F_2 R_2 = 45.216 \times 4 = 180.8 \text{ N cm}$$

Example 1.7

A hydraulic system has 380 L reservoir mounted above the pump to produce a positive pressure (above atmospheric) at the pump inlet, as shown in Fig. 1.10. The purpose of the positive pressure is to prevent the pump from cavitating, when operating, especially at start up. If the pressure at the pump inlet is to be 0.35 bar prior to turning the pump ON and the oil has a specific gravity of 0.9, what should the oil level be above the pump inlet?

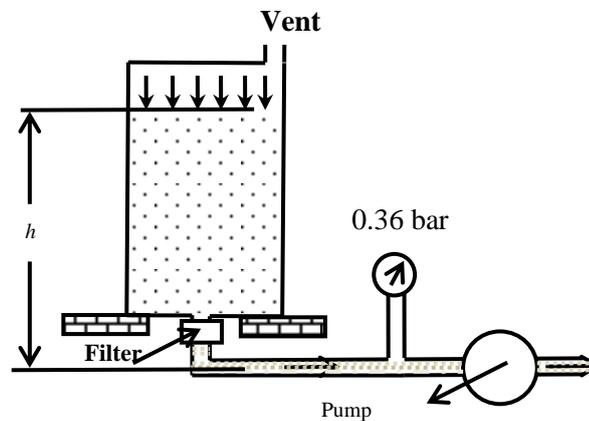


Figure 1.10

We know that

$$p_{\text{oil}} = \gamma_{\text{oil}} H_{\text{oil}}$$

$$\Rightarrow H_{\text{oil}} = \frac{p_{\text{oil}}}{\gamma_{\text{oil}}} = \frac{0.35 \times 10^5 \text{ N/m}^2}{0.90 \times 9797 \text{ N/m}^3} = 3.96 \text{ m}$$

Thus, oil level should be 3.96 m above the pump inlet.

Example 1.8

For the hydraulic pressure shown in Fig. 1.11, what would be the pressure at the pump inlet if the reservoir were located below the pump so that the oil level would be 1.22 m below the pump inlet? The specific gravity of oil is 0.90. Ignore frictional losses and changes in kinetic energy on the pressure at the pump inlet. Would this increase or decrease the chances for having pump cavitation? If yes, why?

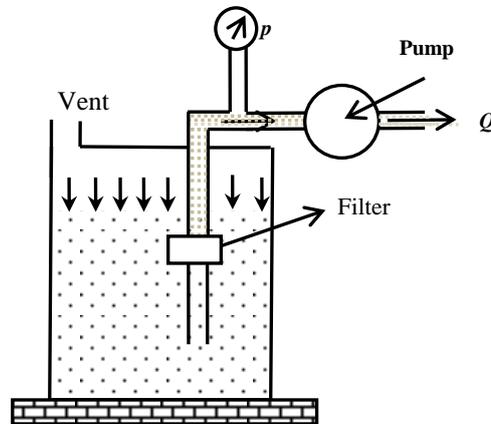


Figure 1.11

Solution: We know that

$$\begin{aligned} p_{\text{oil}} &= -\gamma_{\text{oil}} H_{\text{oil}} \\ &= -0.90 \times 9797 \frac{\text{N}}{\text{m}^3} \times 1.22 \text{ m} = -10757 \text{ Pa} \\ &= -0.10757 \text{ bar (gauge)} \end{aligned}$$

Frictional losses and changes in kinetic energy would cause the pressure at the pump inlet to increase negatively (greater suction pressure) because pressure energy decreases as per Bernoulli's equation. This would increase the chances for having the pump cavitation because the pump inlet pressure more closely approaches the vapor pressure of the fluid (usually about 0.34 bar suction) or -0.34 bar (gauge), allowing for the formation and collapse of vapor bubbles

Example 1.9

A hydraulic cylinder is to compress a body down to bale size in 10 s. The operation requires a 3 m stroke and a 40000 N force. If a 10 MPa pump has been selected, assuming the cylinder to be 100% efficient, find

- The required piston area.
- The necessary pump flow rate.
- The hydraulic power delivered to the cylinder.
- The output power delivered to the load.
- Also solve parts (a)–(d) assuming a 400 N friction force and a leakage of 1 LPM. What is the efficiency of the cylinder with the given friction force and leakage?

Solution:

(a) Since the fluid pressure is undiminished, we have $p_1 = p_2 = 10 \text{ MPa}$. Now

$$p_2 = \frac{F_2}{A_2} \Rightarrow A_2 = \frac{F_2}{p_2} = \frac{40000}{10 \times 10^6} = 0.004 \text{ m}^2$$

which is the required piston area.

(b) Stroke length $l = 3 \text{ m}$, time for stroke $t = 10 \text{ s}$, piston area $A_2 = 0.004 \text{ m}^2$. Flow rate is

$$Q = \frac{A_2 l}{t} = \frac{0.004 \times 3}{10} = 12 \times 10^{-4} (\text{m}^3/\text{s}) = 72 \text{LPM}$$

(c) Power delivered to the cylinder

Power = Pressure \times Flow rate

$$= (10 \times 10^6) \times (12 \times 10^{-4})$$

$$= 12000 \text{ W} = 12 \text{ kW}$$

(d) Power delivered to load is

$$\text{Power} = \frac{F \times l}{t} = \frac{40000 \times 3}{10} = 12000 \text{ W} = 12 \text{ kW}$$

Since efficiency is assumed to be 100%, both powers are the same.

(e) With a friction force of $f = 400 \text{ N}$ and 1LPM leakage, piston area is

$$A'_2 = \frac{F_2 + f}{p_2} = \frac{40000 + 400}{10 \times 10^6} = 0.00404 \text{ m}^2$$

Now pump flow rate is

$$Q' = \frac{A'_2 l}{t} = \frac{0.00404 \times 3}{10} \\ = 12.12 \times 10^{-4} \text{ m}^3 / \text{s} = 72.72 \text{LPM}$$

So

$$\text{Total flow} = Q' + \text{Leakage}$$

$$= 72.72 + 1$$

$$= 73.72 \text{LPM}$$

$$= 12.287 \times 10^{-4} \text{ m}^3 / \text{s}$$

Power delivered to the cylinder is given by

$$p \times Q = (10 \times 10^6) \times (12.287 \times 10^{-4}) \\ = 12287 \text{ W} = 12.287 \text{ kW}$$

Power delivered to the load is

$$\text{Power} = \frac{F \times l}{t} = \frac{40000 \times 3}{10} = 12000 \text{ W} = 12 \text{ kW}$$

It will remain the same as without losses. The efficiency of the cylinder

$$\eta = \frac{\text{Power delivered to load}}{\text{Power delivered to cylinder}} \\ = \frac{12}{12.287} \times 100 = 97.66\%$$

Example 1.10

An automobile lift raises a 15600 N car 2.13 m above the ground floor level. If the hydraulic cylinder contains a piston of diameter 20.32 cm and a rod of diameter 10.16 cm, determine the

- Work necessary to lift the car.
- Required pressure.
- Power if the lift raises the car in 10 s.
- Descending speed of the lift for $0.000629 \text{ m}^3/\text{s}$ flow rate.
- Flow rate for the auto to descend in 10 s.

Solution:

(a) We have

$$\begin{aligned}\text{Work necessary to lift the car} &= \text{Force} \times \text{Distance} \\ &= 15600 \times 2.13 \text{ m} = 33200 \text{ N m}\end{aligned}$$

(b) We have

$$\text{Piston area} = \frac{\pi(0.2032)^2}{4} \text{ m}^2 = 0.0324 \text{ m}^2$$

So required pressure is

$$\begin{aligned}\text{Pressure} &= \frac{\text{Force}}{\text{Area}} = \frac{15600}{0.0324} \\ &= 481000 \text{ N / m}^2 = 481 \text{ kPa}\end{aligned}$$

(c) We have

$$\begin{aligned}\text{Power} &= \frac{\text{Work done}}{\text{Time}} = \frac{33200}{10} \\ &= 3320 \text{ N m / s} = 3320 \text{ W} = 3.32 \text{ kW}\end{aligned}$$

(d) $Q = 0.000629 \text{ m}^3/\text{s}$.

$$\text{Annulus area} = \frac{\pi(0.2032)^2 - \pi(0.1016)^2}{4} = 0.0243 \text{ m}^2$$

So

$$\text{Decending speed of the lift} = \frac{\text{Flow rate}}{\text{Annulus area}} = \frac{0.000629}{0.0243} = 0.0259 \text{ m / s}$$

(e) Flow rate for the auto to descend in 10 s is

$$\begin{aligned}\text{Flow rate} &= \text{Annulus area} \times \frac{\text{Distance}}{\text{Time}} \\ &= 0.0243 \times \frac{2.13}{10} = 0.00518 \text{ m}^3/\text{s}\end{aligned}$$

1.3.2 Practical Applications of Pascal's Law

The practical applications of Pascal's law are numerous. In this section, two applications of Pascal's law are presented: (a) The hand-operated hydraulic jack and (b) the air-to-hydraulic pressure booster.

1.3.2.1 Hand-Operated Hydraulic Jack

This system uses a piston-type hand pump to power a hydraulic load cylinder for lifting loads, as illustrated in Fig. 1.12. The operation is as follows:

1. A hand force is applied at point A of handle ABC which is pivoted at point C. The piston rod of the pump cylinder is pinned to the input handle of the pump piston at point B.
2. The pump cylinder contains a small-diameter piston that is free to move up and down. The piston and rod are rigidly connected together. When the handle is pulled up, the piston rises and creates a vacuum in the space below it. As a result, the atmospheric pressure forces the oil to leave the oil tank and flow through check valve 1 to fill the void created below the pump piston. This is the suction process.
3. A check valve allows flow to pass in only one direction, as indicated by the arrow. When the handle is pushed down, oil is ejected from the small-diameter pump cylinder and it flows through check valve 2 and enters the bottom end of the large-diameter load cylinder.

4. The load cylinder is similar in construction to the pump cylinder and contains a piston connected to a rod. Pressure builds up below the load piston and equals the pressure generated by the pump piston. The pressure generated by the pump piston equals the force applied to the pump piston rod divided by the area of the pump piston.
5. The load that can be lifted equals the product of the pressure and the area of the load piston. Also, each time when the input handle is cycled up and down, a specified volume of oil is ejected from the pump to raise the load cylinder a given distance.
6. The bleed valve is a hand-operated valve, which, when opened, allows the load to be lowered by bleeding oil from the load cylinder back to the oil tank.

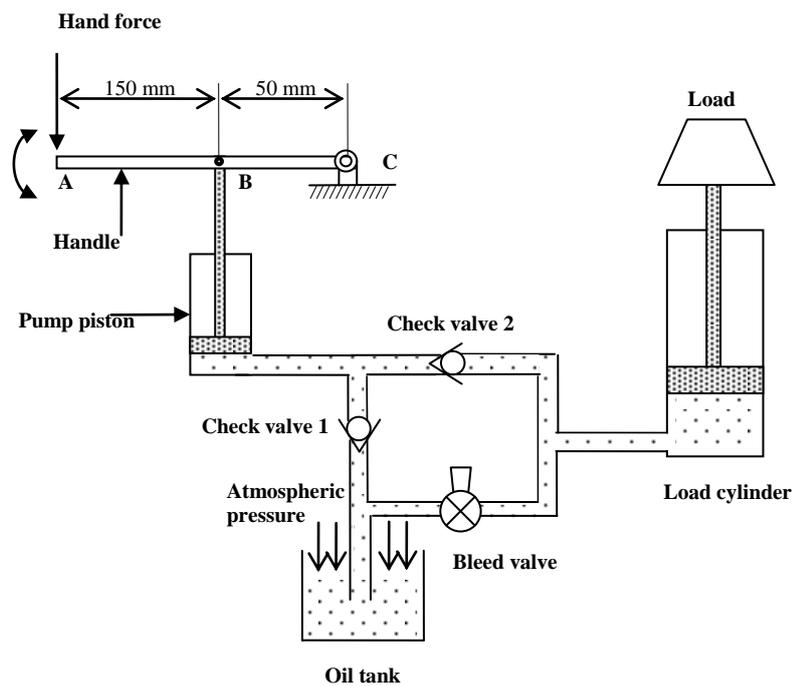


Figure 1.12 Application of Pascal's law: Hand-operated hydraulic jack

1.3.2.2 Air-to-Hydraulic Pressure Booster

This device is used for converting shop air into higher hydraulic pressure needed for operating hydraulic cylinders requiring small to medium volumes of higher pressure oil. It consists of a cylinder containing a large-diameter air piston driving a small-diameter hydraulic piston that is actually a long rod connected to the piston. Any shop equipped with an airline can obtain smooth, efficient hydraulic power from an air-to-hydraulic pressure booster hooked into the air line. The alternative would be a complete hydraulic system including expensive pumps and high-pressure valves. Other benefits include space savings and low operating and maintenance costs.

Figure 1.13 shows an application where an air-to-hydraulic pressure booster supplies high-pressure oil to a hydraulic cylinder whose short stroke piston is used to clamp a workpiece to a machine tool table. Since shop air pressure normally operates at 100 psi, a pneumatically operated clamp would require an excessively large cylinder to rigidly hold the workpiece while it is being machined.

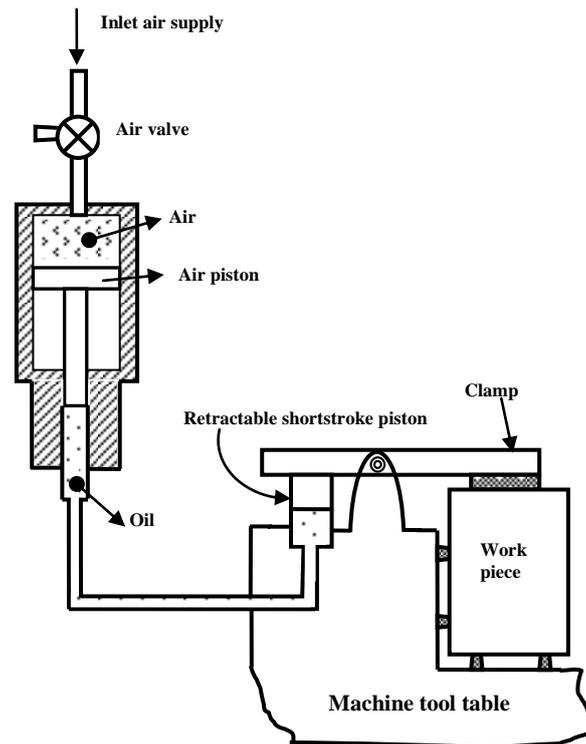


Figure 1.13 Application of Pascal's law: Air-to-hydraulic pressure booster

The air-to-hydraulic pressure booster operates as follows. Let us assume that the air piston has 10 cm^2 area and is subjected to a 10 bar air pressure. This produces a 1000 N force on the booster's hydraulic piston. Thus, if the area of the booster's hydraulic piston is 1 cm^2 , the hydraulic oil pressure is 100 bar. As per Pascal's law, this produces 100 bar oil at the short stroke piston of the hydraulic clamping cylinder mounted on the machine tool table.

The pressure ratio of an air-to-hydraulic pressure booster can be found by using the following equation:

$$\begin{aligned} \text{Pressure ratio} &= \frac{\text{Output oil pressure}}{\text{Input oil pressure}} \\ &= \frac{\text{Area of air piston}}{\text{Area of hydraulic piston}} \end{aligned}$$

Substituting into the above equation for the earlier mentioned pressure booster, we have

$$\text{Pressure ratio} = \frac{10000 \text{ kPa}}{1000 \text{ kPa}} = \frac{10 \text{ cm}^2}{1 \text{ cm}^2}$$

For a clamping cylinder piston area of 0.5 cm^2 , the clamping force equals $1000 \text{ N/cm}^2 \times 0.5 \text{ cm}^2$ or 500 N. To provide the same clamping force of 500 N without booster requires a clamping cylinder piston area of 5 cm^2 , assuming 10 bar air pressure. Air-to-hydraulic pressure boosters are available in a wide range of pressure ratios and can provide hydraulic pressures up to 1000 bar using approximately 7 bar shop air.

Example 1.11

An operator makes 15 complete cycles in 15 s interval using the hand pump shown in Fig. 1.14. Each complete cycle consists of two pump strokes (intake and power). The pump has a piston of diameter 30 mm and the load cylinder has a piston of diameter 150 mm. The average hand force is 100 N during each power stroke.

- (a) How much load can be lifted?
- (b) How many cycles are required to lift the load by 500 mm, assuming no oil leakage? The pump piston has 20 mm stroke.
- (c) What is the output power assuming 80% efficiency?

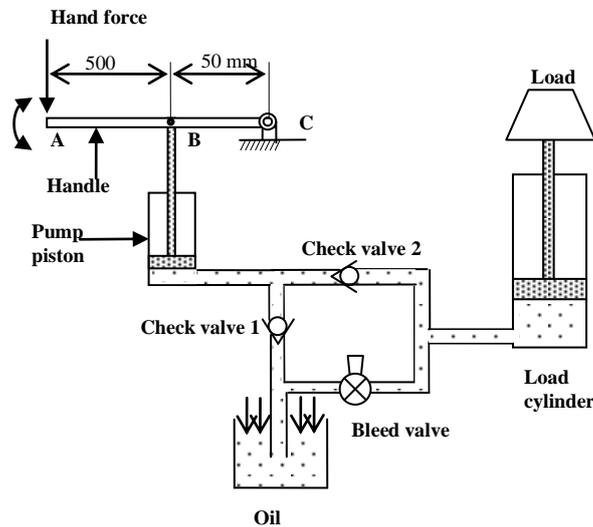


Figure 1.14

Solution: Given: pump diameter $d = 30\text{ mm}$, load cylinder diameter $D = 150\text{ mm}$, hand force $f = 100\text{ N}$, number of cycles $n = 15$ strokes/s, pump piston force

$$F_1 = \frac{100 \times 550}{50} = 1100\text{ N}$$

(a) **Load capacity:** Now since the pressure remains undiminished throughout, we have $p_1 = p_2$. Therefore,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow F_2 = \frac{\pi D^2 / 4}{\pi d^2 / 4} F_1 = \frac{150^2}{30^2} \times 1100 = 27500\text{ N} = 27.5\text{ kN}$$

(b) **Number of cycles:** Stroke length $l = 20\text{ mm}$. Let the number of strokes be N . Then assuming no leakage, we get

$$Q_1 = Q_2$$

where

$$Q_1 = \text{Total volume of fluid displaced by pump piston}$$

$$= (\text{Area} \times \text{Stroke}) \times \text{Number of strokes} = N \times A_1 l$$

$$Q_2 = \text{Flow rate of load cylinder} = (\text{Area} \times \text{Stroke of load cylinder})$$

$$= A_2 \times 500$$

So we get

$$N \times A_1 l = A_2 \times 500$$

$$\Rightarrow N = \frac{150^2}{20 \times 30^2} \times 500 = 625$$

Hence, the number of cycles required is 625.

(c) **Output power:**

$$\text{Input power} = F_1 \times l \times n$$

$$\text{Output power} = \eta \times F_1 \times l \times n = 0.8 \times 1100 \times 0.02 \times 15 = 264 \text{ W}$$

Example 1.12

For the pressure booster of Fig. 1.15, the following data are given:

Inlet oil pressure (p_1) = 1 MPa

Air piston area (A_1) = 0.02 m²

Oil piston area (A_2) = 0.001 m²

Load carrying capacity = 300000 N

Find the load required on load piston area A_3 .

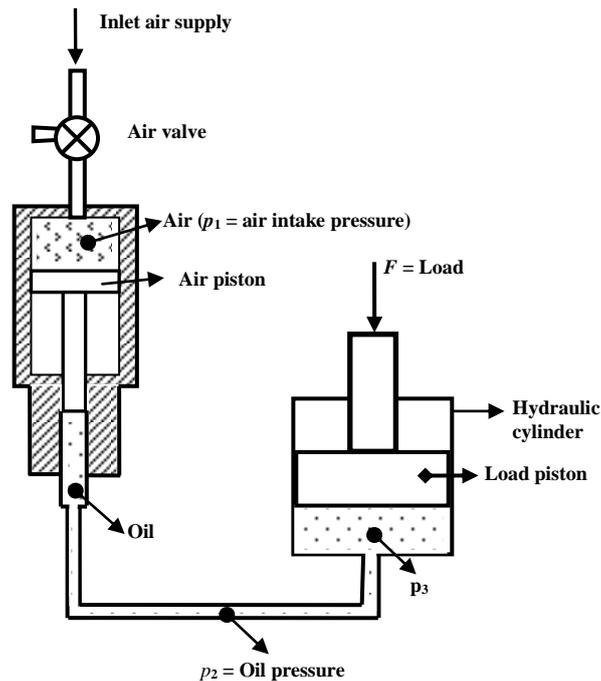


Figure 1.15

Solution: We know that

$$p_2 = \frac{p_1 A_1}{A_2} = \frac{1 \text{ MPa} \times 0.02 \text{ m}^2}{0.001 \text{ m}^2} = 20 \text{ MPa}$$

Also $p_3 = p_2 = 20 \text{ MPa}$. So

$$A_3 = \frac{F}{p_3} = \frac{300000 \text{ N}}{20 \times 10^6 \text{ N/m}^2} = 0.015 \text{ m}^2$$

1.4 Conservation of Energy

The first law of thermodynamics states that energy can neither be created nor be destroyed. Moreover, all forms of energy are equivalent. The various forms of energy present in fluid flow are briefly discussed. The total energy includes potential energy due to elevation and pressure and also kinetic energy due to velocity. Let us discuss all these in detail.

1. Kinetic energy of a flowing fluid: A body of mass m moving with velocity v possesses a kinetic energy (KE), that is,

$$KE = \frac{mv^2}{2}$$

Thus, if a fluid were flowing with all particles moving at the same velocity, its kinetic energy would also be $(1/2)(m)v^2$; this can be written as

$$\frac{KE}{Weight} = \frac{\frac{1}{2}mv^2}{(\gamma)Volume} = \frac{\frac{1}{2}[(\gamma)Volume]v^2}{(\gamma)Volume} = \frac{v^2}{2g}$$

where g is the acceleration due to gravity. In SI units, $v^2/2g$ is expressed as $Nm/N = m$.

2. Potential energy due to elevation (z): Consider a unit weight of fluid as shown in Fig. 1.16. The potential energy of a particle of a fluid depends on its elevation above any arbitrary plane. We are usually interested only in the differences of elevation, and therefore the location of the datum plane is determined solely by consideration of convenience. A fluid particle of weight W situated at a distance Z above datum possesses a potential energy Wz . Thus, in SI units, its potential energy per unit weight is expressed as $Nm/N = m$.

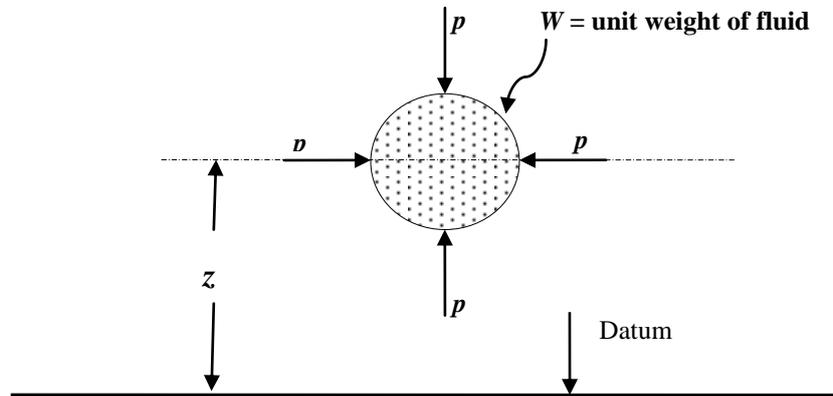


Figure 1.16 Potential energy due to elevation

3. Potential energy due to pressure (PE): This term represents the energy possessed by a fluid per unit weight of fluid by virtue of the pressure under which the fluid exists:

$$PE = \frac{p}{\gamma}$$

where γ is the specific weight of the fluid. PE has the unit of meter. The total energy possessed by the weight of fluid remains constant (unless energy is added to the fluid via pumps or removed from the fluid via hydraulic motors or friction) as the weight W flows through a pipeline of a hydraulic system. Mathematically, we have

$$E_{Total} = z + \frac{p}{\gamma} + \frac{v^2}{2g}$$

Energy can be changed from one form to another. For example, the chunk of fluid may lose elevation as it flows through a hydraulic system and thus has less potential energy. This, however, would result in an equal increase in either the fluid's pressure energy or its kinetic energy. The energy equation takes into account the fact that energy is added to the fluid via pumps and that energy is removed from the fluid via hydraulic motors and friction as the fluid flows through actual hydraulic systems.

Example 1.13

Oil with specific gravity 0.9 enters a tee, as shown in Fig. 1.18, with velocity $v_1 = 5$ m/s. The diameter at section 1 is 10 cm, the diameter at section 2 is 7 cm and the diameter at section 3 is 6 cm. If equal flow rates are to occur at sections 2 and 3, find the velocities v_2 and v_3 .

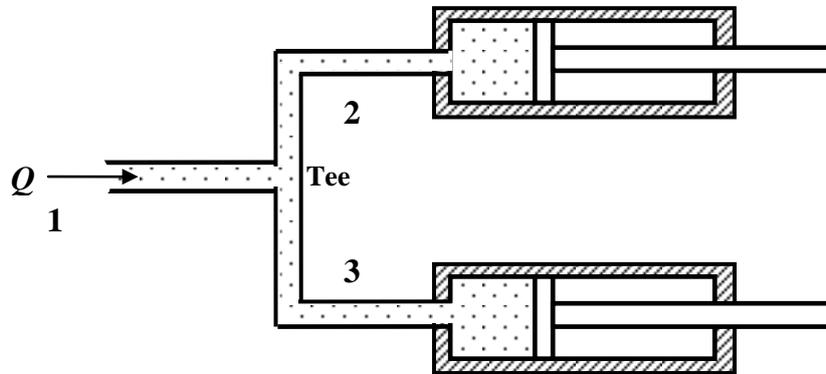


Figure 1.18

Solution: Assuming no leakage

$$Q_1 = Q_2 + Q_3$$

Also,

$$Q_2 = Q_3 = \frac{1}{2} Q_1 = \frac{1}{2} A_1 v_1$$

$$= \frac{1}{2} \times \frac{\pi d_1^2}{4} \times v_1 = \frac{1}{2} \times \frac{\pi \times 0.1^2}{4} \times 5 = 19.63 \times 10^{-3} \text{ m}^3/\text{s}$$

Therefore,

$$v_2 = \frac{Q_2}{A_2} = \frac{19.63 \times 10^{-3}}{\pi \times 0.07^2 / 4} = 5.1 \text{ m/s}$$

$$v_3 = \frac{Q_3}{A_3} = \frac{19.63 \times 10^{-3}}{\pi \times 0.06^2 / 4} = 6.942 \text{ m/s}$$

Example 1.14

A double-rod cylinder is one in which a rod extends out of the cylinder at both ends (Fig. 1.19). Such a cylinder with a piston of diameter 75 mm and a rod of diameter 50 mm cycles through 254 mm stroke at 60 cycles/min. What LPM size pump is required?

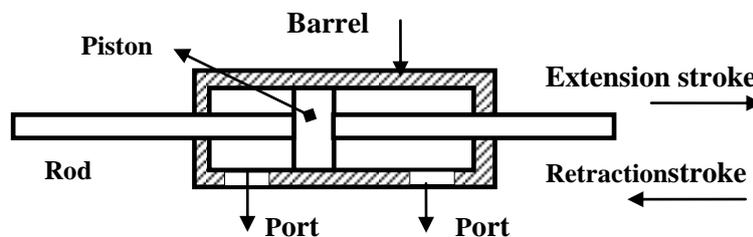


Figure 1.19

Solution: The annulus area is

$$A_{\text{Annulus}} = \frac{\pi(75^2 - 50^2)}{4} = 2454 \text{ mm}^2$$

Volume of oil displaced per minute (m^3/s) is

Area \times Stroke length \times No. of cycles per second

Now

$$Q = \frac{\pi(75^2 - 50^2)\text{mm}^2}{4} \times 10^{-6} \text{ m}^2 \times \left\{ \frac{254}{1000} \times 2 \right\} (\text{m}) \times \frac{60}{60} (\text{s})$$

$$= 0.001296 \text{ m}^3/\text{s} = 77.8 \text{ LPM}$$

We can select 80 LPM pump.

Example 1.15

A cylinder with a piston of diameter 8 cm and a rod of diameter 3 cm receives fluid at 30 LPM. If the cylinder has a stroke of 35 cm, what is the maximum cycle rate that can be accomplished?

Solution: We know that

Volume of oil displaced per minute (m^3/min) = Area \times Stroke length \times No. of cycles per minute

So

$$Q = \frac{\pi(0.08^2)\text{m}^2}{4} \times \left\{ \frac{35}{100} \right\} (\text{m}) \times N (\text{cycles}/\text{min}) + \frac{\pi(0.08^2 - 0.03^2)\text{m}^2}{4} \times \left\{ \frac{35}{100} \right\} (\text{m}) \times N (\text{cycles}/\text{min})$$

$$= 0.03 \text{ m}^3/\text{min}$$

$$\Rightarrow 0.030 = 0.00176 + 0.0015 \times N$$

$$\Rightarrow N = 9.2 \text{ cycles}/\text{min}$$

Example 1.16

A hydraulic pump delivers a fluid at 50 LPM and 10000 kPa. How much hydraulic power does the pump produce?

Solution: We have

$$Q = 50 \text{ LPM} = \frac{50}{60 \times 10^3} = 0.833 \times 10^{-3} \text{ m}^3/\text{s}$$

Now

$$1 \text{ L} = 1000 \text{ cc} = 1000 \times 10^{-6} \text{ m}^3 = 10^{-3} \text{ m}^3$$

So

$$\text{Power (kW)} = p (\text{kPa}) \times Q (\text{m}^3/\text{s})$$

$$= 10000 \times 0.833 \times 10^{-3}$$

$$= 8.33 \text{ kW} = 8330 \text{ W}$$

1.7 The Energy Equation

The Bernoulli equation discussed above can be modified to account for fractional losses (H_L) between stations 1 and 2. Here H_L represents the energy loss due to friction of 1 kg of fluid moving from station 1 to station 2. As discussed earlier, H_p represents the energy head put into the flow by the pump. If there exists a hydraulic motor or turbine between stations 1 and 2, then it removes energy from the fluid. If H_m (motor head) represents the energy per kg of fluid removed by a hydraulic motor, the modified Bernoulli equation (also called the energy equation) is stated as follows for a fluid flowing in a pipeline from station 1 to station 2: The total energy possessed by 1 kg of fluid at station 1 plus the energy added to it by a pump minus the energy removed from it by a hydraulic motor minus the energy it loses due to friction equals the total energy possessed by 1 kg of fluid when it arrives at station 2. The energy equation is as follows, where each term represents a head and thus has the unit of length:

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

1.9 Elements of Hydraulic Systems and the Corresponding Bernoulli's Equation

The main elements of hydraulic systems are pump, motor, pipes, valves and fittings. Let us write the energy flow from point 1 to point 2 as shown in Fig. 1.22. After the fluid leaves point 1, it enters the pump where energy is added. A prime mover, such as an electric motor, drives the pump and the impeller of the pump transfers the energy to the fluid. Then the fluid flows through a piping system composed of a valve, elbows and the lengths of pipe in which energy is dissipated from the fluid and is lost. Before reaching point 2, the fluid flows through a fluid motor that removes some of the energy to drive an external device. The general energy equation accounts for all these energies.

In a particular problem, it is possible that not all of the terms in the general energy equation are required. For example, if there is no mechanical device between the sections of interest, the terms H_p and H_m will be zero and can be left out of the equation. If energy losses are so small that they can be neglected, the term H_L can be left out. If both these conditions exist, it can be seen that the energy equation reduces to Bernoulli's equation.

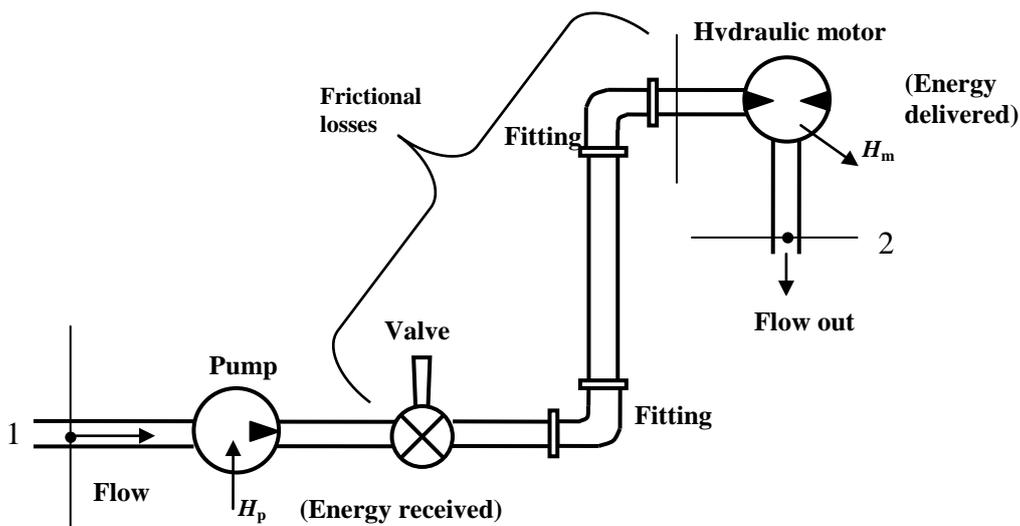


Figure 1.22 Elements of a hydraulic system

Example 1.17

(a) Calculate the work required for a pump to pump water from a well to ground level 125 m above the bottom of the well (see Fig.1.23). At the inlet to the pump, the pressure is 96.5 kPa, and at the system outlet, it is 103.4 kPa. Assume the constant pipe diameter. Use $\gamma = 9810 \text{ N/m}^3$, and assume it to be constant. Neglect any flow losses in the system.

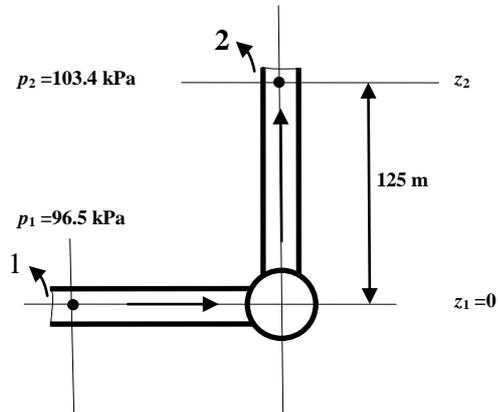


Figure 1.23

Given $z_1 = 0$, $z_2 = 125 \text{ m}$, $p_1 = 96.5 \text{ kPa}$, $p_2 = 103.4 \text{ kPa}$, $H_L = 0$, $D_1 = D_2$. Find H_p .

Assumptions: Steady incompressible flow, no losses

Basic equations:

Continuity: $A_1 v_1 = A_2 v_2$

$$\text{Energy equation: } \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + H_L$$

(b) Solve the above problem if there is friction in the system whose total head loss equals 12.5 m.

Given $z_1 = 0$, $z_2 = 125 \text{ m}$, $p_1 = 96.5 \text{ kPa}$, $p_2 = 103.4 \text{ kPa}$, $H_L = 12.5 \text{ m}$, $D_1 = D_2$. Find H_p .

Assumptions: Steady incompressible flow, no losses

Basic equations:

Continuity: $A_1 v_1 = A_2 v_2$

$$\text{Energy equation: } \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + H_L$$

Solution:

(a) Write the energy equation

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + H_L$$

Note that $v_1 = v_2$ and $H_L = 0$. Thus,

$$H_p = \frac{p_2}{\gamma} - \frac{p_1}{\gamma} + z_2 - z_1$$

With $z_1 = 0$ we get

$$H_p = \frac{103.4 \text{ kPa}}{9.81 \text{ kN/m}^3} - \frac{96.5 \text{ kPa}}{9.81 \text{ kN/m}^3} + 125 = 125.7 \text{ m}$$

(b) Write the energy equation

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + H_L$$

As before, $v_1 = v_2$ but $H_L = 12.5$ m. Therefore,

$$H_p = \frac{p_2}{\gamma} - \frac{p_1}{\gamma} + z_2 - z_1 + H_L$$

With $z_1 = 0$ we get

$$H_p = \frac{103.4 \text{ kPa}}{9.81 \text{ kN/m}^3} - \frac{96.5 \text{ kPa}}{9.81 \text{ kN/m}^3} + 137.5 = 138.2 \text{ m}$$

Note that the pump is required to overcome the additional friction head loss, and for the same flow, this requires more pump work. The additional pump work is equal to the head loss.

Example 1.18

A hydraulic turbine is connected as shown in Fig. 1.24. How much power will it develop? Use 1000 kg/m^3 for the density of water. Neglect the flow losses in the system.

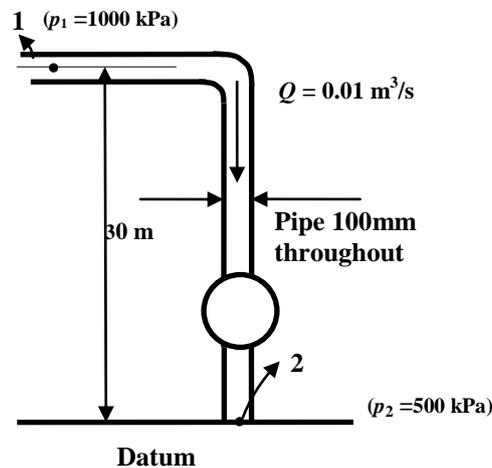


Figure 1.24

Given $z_1 = 30$ m, $z_2 = 0$, $p_1 = 1000$ kPa, $p_2 = 500$ kPa, $H_L = 0$, $D_1 = D_2 = 100$ mm, $Q = 0.01 \text{ m}^3/\text{s}$.

Find turbine power.

Assumptions: Steady incompressible flow, no losses

Basic equations:

Continuity: $A_1 v_1 = A_2 v_2$

$$\text{Energy: } \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + H_L$$

$$\text{Power: } P = H_T \times Q \times \gamma$$

Solution: Again let us write the energy equation, but this time for a turbine:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + H_L$$

Since there are no losses in the pipe and the pipe diameter is constant, $v_1 = v_2$, $z_2 = 0$, $z_1 = 30$ m.

Therefore, H_T is found as

$$H_T = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2)$$

Using the data given we get

$$H_T = \frac{(1000 - 500)1000}{1000 \times 9.81} + (30 - 0) = 80.97 \text{ m}$$

Horsepower is given by

$$\begin{aligned} \text{Horsepower} &= H_T \times Q \times \gamma \\ &= 80.97 \times 0.01 \times 1000 \times 9.81 \\ &= 7941 \text{ W} = 7.941 \text{ kW} \end{aligned}$$

Example 1.19

For the hydraulic system shown in Fig.1.25, the following data are given:

The pump is adding 4 kW to the fluid (i.e., the hydraulic power of the pump).

The pump flow is $0.002 \text{ m}^3/\text{s}$.

The pipe has an inside diameter of 25 mm.

The specific gravity of oil is 0.9.

Point 2 is at an elevation of 0.6 m above the oil level, that is, point 1.

The head loss due to friction in the line between points 1 and 2 is 10.

Determine the fluid pressure at point 2, the inlet to the hydraulic motor. Neglect the pressure drop at the strainer. The oil tank is vented to atmosphere.

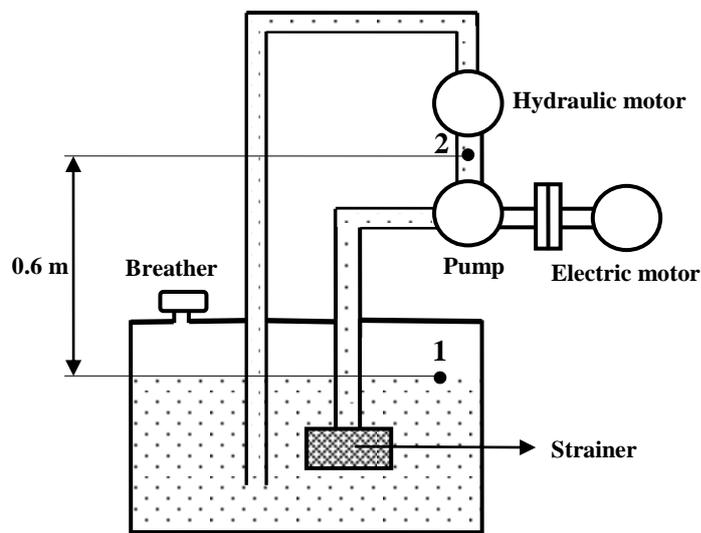


Figure 1.25

Solution: Given $p_1 = 0$ (as the tank is vented to the atmosphere)

$$P \text{ (kW)} = 4 \text{ (kW)} = 4 \times 10^3 \text{ W}$$

$$Q = 0.002 \text{ m}^3/\text{s}$$

$$D_p = 25 \text{ mm} = 0.025 \text{ m}$$

$$SG = 0.9$$

$$z_2 - z_1 = 6 \text{ m}$$

$$H_L = 10 \text{ m}$$

$$H_m = 0 \text{ (there is no motor between 1 and 2)}$$

The problem can be solved by using the energy equation (Bernoulli's equation):

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = z_2 + \frac{P_2}{\gamma} + \frac{v_2^2}{2g}$$

We can take $v_1 = 0$ since the tank cross-section is large. Let us compute some of the unknown terms in the equation. The pump head is given by

$$H_p = \frac{P \text{ (W)}}{\gamma \text{ (N/m}^3\text{)} \times Q \text{ (m}^3\text{/s)}}$$

$$= \frac{4 \times 10^3}{0.9 \times 9800 \times 0.002} = 226.7 \text{ m}$$

The velocity head is

$$v_2 = \frac{Q \text{ (m}^3\text{/s)}}{A \text{ (m}^2\text{)}} = \frac{0.002}{\frac{\pi}{4}(0.025^2)} = 4.07 \text{ m/s}$$

The velocity head is

$$\frac{v_2^2}{2g} = \frac{4.07^2}{2 \times 9.81} = 0.85 \text{ m}$$

Substituting the values into the energy equation and rearranging, we can write

$$z_1 + 0 + 0 + 266.7 - 0 - 10 = z_2 + \frac{p_2}{\gamma} + 0.85$$

$$\Rightarrow \frac{p_2}{\gamma} = (z_1 - z_2) + 266.7 - 10 - 0.85 = -6 + 266.7 - 10 - 0.85$$

$$\Rightarrow \frac{p_2}{\gamma} = 209.85 \text{ m}$$

$$\Rightarrow p_2 = 209.85 \times 0.9 \times 9800 = 1850877 \text{ Pa} = 1850.9 \text{ kPa}$$

Example 1.20

The oil tank for the hydraulic system shown in Fig.1.26 is pressurized at 68 kPa gauge pressure. The inlet to the pump is 3 m below the oil level. The pump flow rate is 0.001896 m³/s. Find the pressure at station 2. The specific gravity of oil is 0.9 and kinematic viscosity of oil is 100 cS. Assume the pressure drop across the strainer to be 6.9 kPa. Also given the pipe diameter is 38 mm and the total length of the pipe is 6 m.

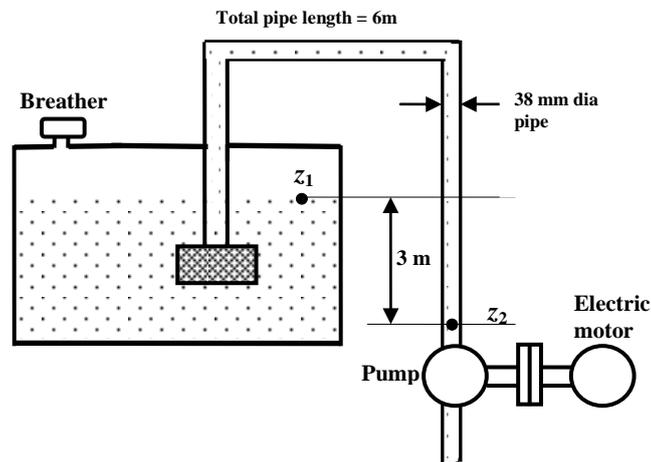


Figure 1.26

Solution: We have $p_1 = 68 \text{ kPa}$, $z_1 - z_2 = 3 \text{ m}$, $Q = 0.001896 \text{ m}^3\text{/s}$, $p_s = 6.9 \text{ kPa}$, $SG = 0.9$, $D_p = 38 \text{ mm}$, $\nu = 100 \text{ cS}$, $L_p = 6 \text{ m}$. To calculate p_2 .

By the application of Bernoulli's (energy) equation, we can write

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Now $z_1 - z_2 = 3\text{ m}$, $H_m = 0$ (because there is no fluid motor between points 1 and 2), $v_1 = 0$ (assuming the oil tank area to be large). The velocity at point 2, v_2 , is

$$v_2 = \frac{Q \text{ (m}^3\text{/s)}}{A \text{ (m}^2)} = \frac{0.001896}{\pi(0.038^2)/4} = 1.67 \text{ m/s}$$

Equivalent velocity head is

$$\frac{v_2^2}{2g} = \frac{1.67^2}{2 \times 9.81} = 0.142 \text{ m}$$

The head loss is

$$H_L = \frac{f \cdot L_p}{D_p} \times \frac{v^2}{2g}$$

Here

$$L_p = \text{Total length of pipe} = 6 \text{ m}$$

$$D_p = \text{Diameter of pipe} = 0.038 \text{ m}$$

Value of f (friction factor) depends on the value of Reynolds number.

$$\begin{aligned} \text{Re} &= \frac{vD_p}{\nu} = \frac{1.67 \times 0.038}{100 \times 10^{-6}} \\ &= 634.6 < 2000, \text{ flow is laminar} \end{aligned}$$

Now

$$f = \frac{64}{\text{Re}} = \frac{64}{634.6} = 0.1$$

So head loss due to friction is

$$H_L = \frac{0.1 \times 6}{0.038} \times 0.142 = 2.24 \text{ m}$$

Case 1: Point 2 is before the pump

When point 2 is before the pump, the pump head is zero, that is, $H_p = 0$. Rearranging the energy equation to solve for the present head, we can write

$$\begin{aligned} \frac{p_2}{\gamma} &= (z_1 - z_2) + \frac{p_1}{\gamma} + H_p - H_L - \frac{v_2^2}{2g} \\ &= 3 + \frac{68000}{0.9 \times 9800} + 0 - 2.24 - 0.142 = 8.33 \text{ m} \end{aligned}$$

$$\Rightarrow p_2 = 8.33 \times 0.9 \times 9800 = 73470 \text{ Pa} = 73.5 \text{ kPa}$$

This value of p_2 is without considering the pressure drop across the strainer. The pressure drop is 6.9 kPa across the strainer. Therefore, the pressure at point 2 is

$$p_{2\text{-actual}} = 73.5 - 6.9 \text{ kPa} = 66.6 \text{ kPa}$$

Which is less than 1 atmospheric pressure (101 kPa)?

Case 2: Point 2 is after the pump

When point 2 is after the pump, the pump head must be taken into account

$$H_p = \frac{P(W)}{\gamma \text{ (N/m}^3) \times Q \text{ (m}^3\text{/s)}}$$

Now

$$\gamma = \text{SG} \times \gamma_{\text{water}} = 0.9 \times 9800 \text{ N/m}^3$$

Also

$$\text{Power} = p_1 \times Q$$

$$P = 68 \left(\frac{\text{kN}}{\text{m}^3} \right) \times 0.001896 \left(\frac{\text{m}^3}{\text{s}} \right) = 0.13 \text{ kW}$$

So

$$H_p = \frac{0.13 \times 10^3 \text{ W}}{0.9 \times 9800 \left(\frac{\text{N}}{\text{m}^3} \right) \times 0.001896 \left(\frac{\text{m}^3}{\text{s}} \right)} = 7.7 \text{ m}$$

Rearranging the energy equation to solve for the present head, we can write

$$\begin{aligned} \frac{p_2}{\gamma} &= (z_1 - z_2) + \frac{p_1}{\gamma} + H_p - \frac{v_2^2}{2g} \\ &= 3 + \frac{68000}{0.9 \times 9800} + 7.7 - 2.24 - 0.142 \\ &= 16.028 \text{ m} \end{aligned}$$

This value of p_2 is without considering the pressure drop across the strainer. The pressure drop is 6.9 kPa across the strainer. Therefore, the pressure at point 2 is

$$p_{2\text{-actual}} = p_2 - 6.9 \text{ kPa} = 141.4 - 6 = 134.5 \text{ kPa}$$

which is greater than 1 atmospheric pressure (101 kPa).

Example 1.21

The volume flow rate through the pump shown in Fig. 1.27 is $7.8 \text{ m}^3/\text{s}$. The fluid being pumped is oil with specific gravity 0.86. Calculate the energy delivered by the pump to the oil per unit weight of oil flowing in the system. Energy losses in the system are caused by the check valve and friction losses as the fluid flows through the piping. The magnitude of such losses has been determined to be 1.86 N m/N .

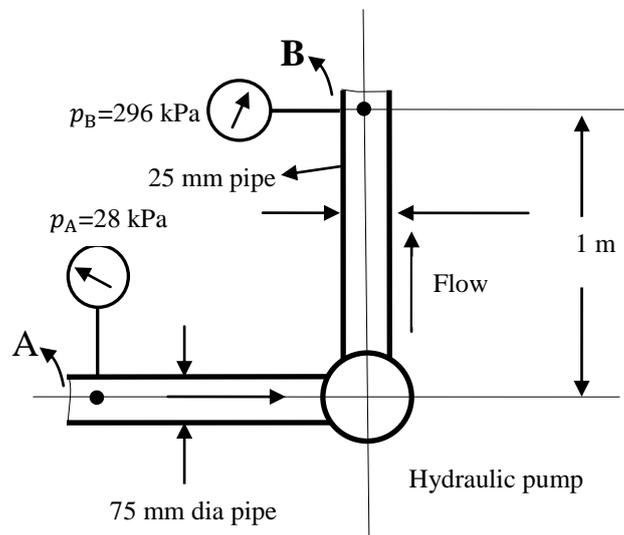


Figure 1.27

Solution: Using the section where pressure gauges are located as the section of interest, write the energy equation for the system, including only the necessary terms:

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} + H_{\text{Added}} - H_{\text{Removed}} - H_{\text{Losses}} = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

or

$$H_{\text{Added}} = \frac{p_B - p_A}{\gamma} + z_B - z_A + \frac{v_B^2 - v_A^2}{2g} + H_{\text{Losses}}$$

In this case, the specific gravity of oil is

$$\gamma = (\text{SG})(\gamma_{\text{water}}) = (0.86)(9.81) = 8.44 \text{ kN/m}^3$$

Since $p_B = 296 \text{ kPa}$ and $p_A = -28 \text{ kPa}$ we get

$$\frac{p_B - p_A}{\gamma} = \frac{[296 - (-28)] \text{ kN}}{\text{m}^2} \times \frac{\text{m}^3}{8.44 \text{ kN}} = 38.4 \text{ m}$$

Now $z_B - z_A = 1 \text{ m}$ as B is at a higher elevation than A. The volume flow rate and continuity equation are used to determine the velocity. Now

$$Q = Av = A_A v_A = A_B v_B$$

$$\Rightarrow v_A = \frac{Q}{A_A} = \left(\frac{0.014 \text{ m}^3}{\text{s}} \right) (4.768 \times 10^{-3}) \text{ m}^2 = 2.94 \text{ m/s}$$

and

$$v_B = \frac{Q}{A_B} = \left(\frac{0.014 \text{ m}^3}{\text{s}} \right) (2.168 \times 10^{-3}) \text{ m}^2 = 6.46 \text{ m/s}$$

So,

$$\frac{v_B^2 - v_A^2}{2g} = \frac{(6.46^2 - 2.94^2) \text{ m}^2 / \text{s}^2}{2 \left(\frac{9.81 \text{ m}}{\text{s}^2} \right)} = 1.69 \text{ m}$$

Given $H_{\text{Losses}} = 1.86 \text{ m}$. Therefore,

$$H_{\text{Added}} = 38.4 \text{ m} + 1.0 \text{ m} + 1.69 \text{ m} + 1.86 \text{ m} = 42.9 \text{ m or } 42.9 \text{ N m/N}$$

That is, the pump delivers 42.9 N m of energy to each newton of oil flowing through it.

Example 1.22

For the hydraulic system of Fig.1.28, the following data are given:

1. Pump flow is $0.001896 \text{ m}^3/\text{s}$.
2. The air pressure at station 1 in the hydraulic tank is 68.97 kPa gauge pressure.
3. The inlet line to the pump is 3.048 m below the oil level.
4. The pipe has an inside diameter of 0.0381 m .

Find the pressure at station 2 if

- (a) There is no head loss between stations 1 and 2.
- (b) There is 7.622 m head loss between stations 1 and 2.

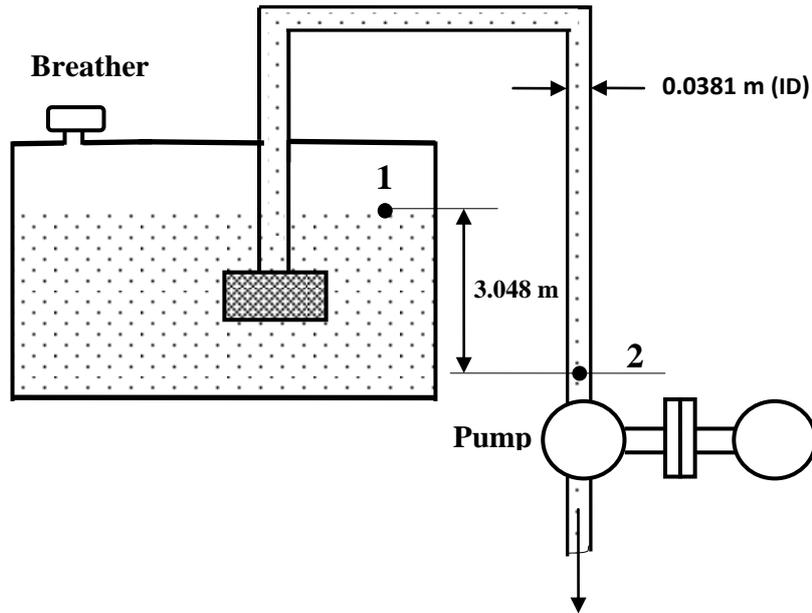


Figure 1.28

Solution: We use Bernoulli's equation

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Case (a): There is no head loss between stations 1 and 2

Here $z_1 - z_2 = 3.0489$ m, $H_L = 0$, $p_1 = 68.97$ kPa. Now

$$v_2 = \frac{\text{Flow}}{\text{Area}} = \frac{0.001896}{\frac{\pi}{4}(0.0381^2)} = 1.66 \text{ m/s}$$

Since there is no pump between 1 and 2, $H_p = 0$.

Since there is no motor between 1 and 2, $H_m = 0$.

Assume $v_1 = 0$ (assuming that area of cross-section is large).

Simplification gives for no head loss,

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = z_2 - z_1 + \frac{v_2^2}{2g}$$

Assuming $\rho g = 8817 \text{ N/m}^3$ we get

$$\frac{68970 \text{ N/m}^2}{8817 \text{ N/m}^3} + 0 + z_1 + 0 - 0 - 0 = \frac{p_2}{\rho g} + \frac{(1.66 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} + z_2$$

Knowing that $z_1 - z_2 = 3.048$ m we get

$$\frac{p_2}{\rho g} = 3.048 + \frac{68970}{8817} - \frac{(1.66)^2}{2 \times 9.81} = 3.048 + 7.82 - 0.142 = 10.73 \text{ m}$$

$$\begin{aligned} p_2 &= 10.73 \text{ (m)} \times 8817 \text{ (N/m}^3) = 94610 \text{ Pa} \\ &= 94.6 \text{ kPa} \end{aligned}$$

Case (b): There is 7.622 m head loss between stations 1 and 2

$$\frac{p_2}{\rho g} = 10.73 \text{ m} - 7.622 = 3.11 \text{ m}$$

So

$$p_2 = 3.11 \text{ (m)} \times 8817 \text{ (N/m}^3\text{)} = 27400 \text{ Pa}$$

$$= 27.4 \text{ kPa}$$

Example 1.23

For the pump in Fig.1.29, $Q_{\text{out}} = 0.00190 \text{ m}^3/\text{s}$ of oil having a specific gravity of 0.9. What is Q_{in} ? Find the pressure difference between A and B if

- (a) The pump is turned OFF.
- (b) The input power to the pump is 1494 W.

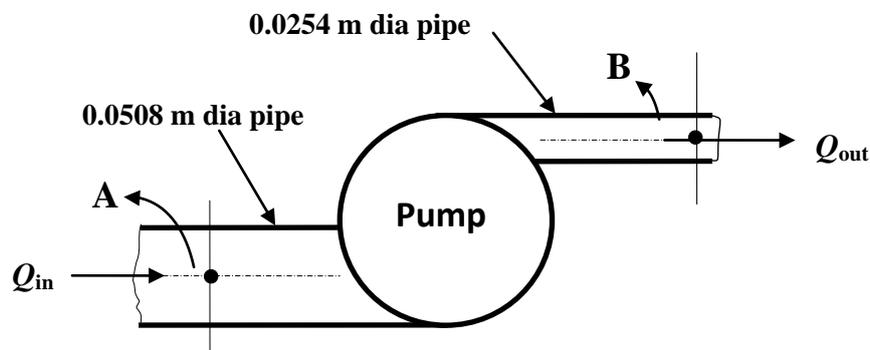


Figure 1.29

Solution:

(a) The pump is turned OFF:

As per Bernoulli's equation, $p_B - p_A = 0$

(b) The input power to the pump is 1494 W

We use Bernoulli's equation:

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A + H_p - H_m - H_L = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

Here

$$H_p = \frac{\text{Pump power (W)}}{\gamma \text{ (N/m}^3\text{)} \times Q \text{ (m}^3\text{/s)}}$$

$$= \frac{1494}{0.9 \times 9800 \times 0.00190} = 89.2 \text{ m}$$

$$v_A = \frac{\text{Flow}}{\text{Area}} = \frac{0.00190}{\frac{\pi}{4} (0.0508^2)} = 0.937 \text{ m/s}$$

$$v_B = \frac{\text{Flow}}{\text{Area}} = \frac{0.00190}{\frac{\pi}{4}(0.0254^2)} = 3.75 \text{ m/s}$$

Substituting values, we have

$$\begin{aligned} \frac{(p_A - p_B)}{\gamma} &= H_p - \frac{(v_B^2 - v_A^2)}{2g} \\ &= 89.2 - \frac{(3.75^2 - 0.937^2)}{2 \times 9.81} \\ &= 88.5 \text{ m} \end{aligned}$$

So

$$\begin{aligned} p_B &= 88.5 \text{ (m)} \times 9800 \text{ (N/m}^3\text{)} \times 0.9 \\ &= 781000 \text{ Pa} \\ &= 781 \text{ kPa} \end{aligned}$$

1.10 Torricelli's Theorem

Torricelli's theorem is Bernoulli's equation with certain assumptions made. Torricelli's theorem states that the velocity of the water jet of liquid is directly proportional to the square root of the head of the liquid producing it. This deals with the setup where there is a large tank with a narrow opening allowing the liquid to flow out (Fig. 1.30). Both the tank and the narrow opening (nozzle) are open to the atmosphere:

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

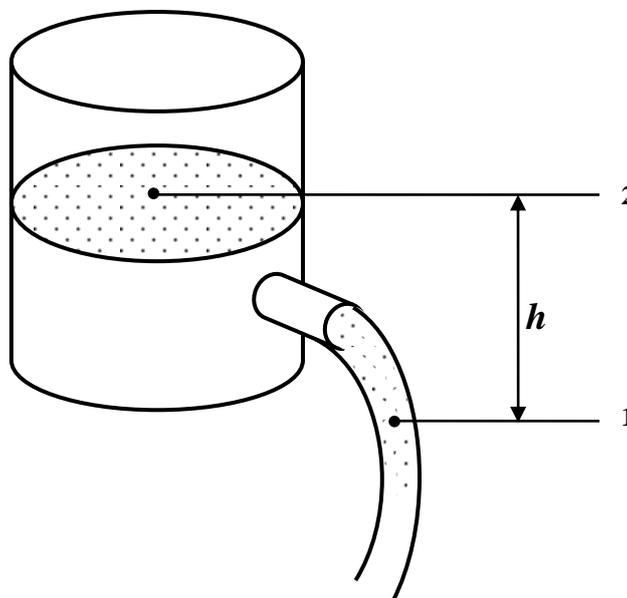


Figure 1.30 Tank with a narrow opening (nozzle)

In this setup, certain assumptions are made:

1. Pressure is the same because the tank and the nozzle are open to the atmosphere, that is, $p_1 = p_2$.
2. Also, let $z_2 - z_1 = h$.
3. The fluid velocity of the tank (water level) is very much slower than the fluid velocity of the nozzle as the area of the liquid surface is much larger than that of the cross section of nozzle, that is, $v_2 \ll v_1$.
4. There is no pump or motor, that is, $H_p = H_m = 0$.
5. There are no frictional losses, that is, $H_L = 0$.

Keeping all these assumptions in mind, Bernoulli's equation gets reduced to

$$v_2 = \sqrt{2gh}$$

where v_2 is the jet velocity (m/s), g is the acceleration due to gravity (m/s^2) and h is the pressure head (m). Now if we do not consider an ideal fluid, then the friction head will be present (H_L). In that case

$$v_2 = \sqrt{2g(h - H_L)}$$

This shows that the velocity of jet decreases if the friction losses are taken into account.

1.11 Siphon

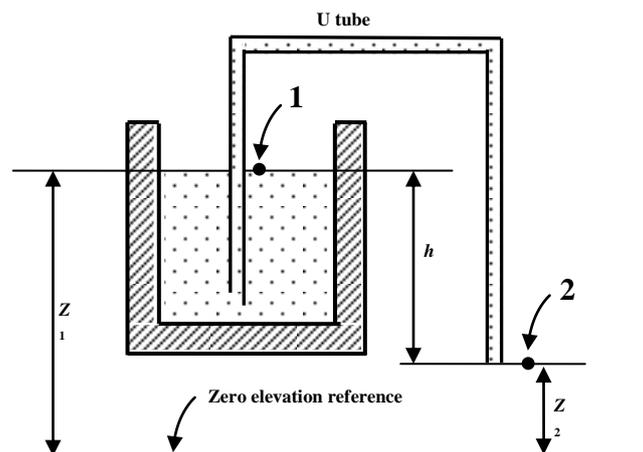


Figure 1.31 The siphon principle

A siphon is a familiar hydraulic device (Fig. 1.31). It is commonly used to cause a liquid to flow from one container in an upward direction over an obstacle to a second lower container in a downward direction. As shown in Fig. 1.31, a siphon consists of a U-tube with one end submerged below the level of the liquid surface, and the free end lying below it on the outside of the container. For the fluid to flow out of the free end, two conditions must be met:

1. The elevation of the free end must be lower than the elevation of the liquid surface inside the container.
2. The fluid must initially be forced to flow up from the container into the center portion of the U-tube. This is normally done by temporarily providing a suction pressure at the free end of the siphon. For example, when siphoning gasoline from an automobile gas tank, a person can develop this suction by momentarily sucking the free end of the hose. This allows atmospheric pressure in the tank to push the gasoline up the U-tube hose, as required. For continuous flow operation, the free end of the U-tube hose must lie below the gasoline level in the tank.

We can analyze the flow through a siphon by applying the energy equation between points 1 and 2 as shown in Fig. 1.31:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + H_p - H_m - H_L = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

The following conditions apply for a siphon:

1. $p_1 = p_2 =$ atmospheric pressure.
2. The area of the surface of the liquid in the container is large so that the velocity v_1 equals essentially 0.

Example 1.24

For the siphon system shown in Fig.1.32, the following data are given: $z_1 = 4$ m, $z_2 = 0.2$ m, $H_L = 0.5$ m. If the inside diameter of the siphon pipe is 30 mm, determine the velocity of the fluid and the flow rate (in LPM) through the siphon. Apply the energy equation and solve the problem.

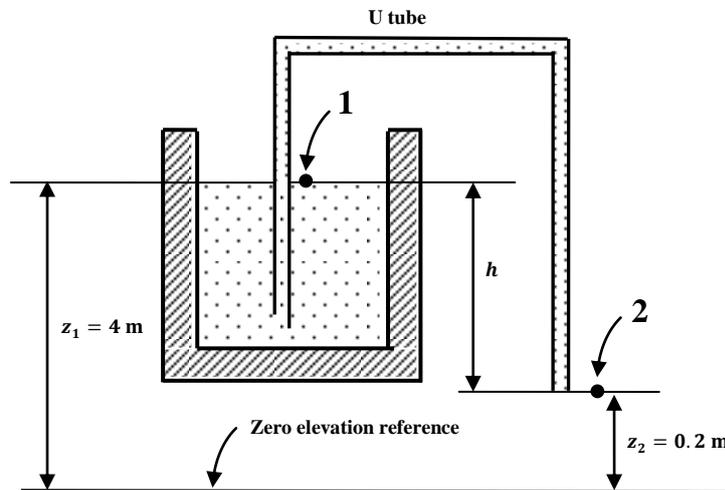


Figure 1.32

Solution: Given $z_1 = 4$ m, $z_2 = 0.2$ m, $H_L = 0.5$ m, $D = 30$ mm = 30×10^{-3} m. To calculate v_2 and Q_2 . This problem can be solved by using the energy equation (modified Bernoulli's theorem) to points (1) and (2) as below:

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

where $p_1 = p_2 = 0$ (atmospheric pressure), $v_1 = 0$ (as the tank is quite large, the velocity is negligible), $H_p = 0$ (no pump), $H_m = 0$ (no motor), $z_1 - z_2 = h$ (the head). Substituting these values in the above equation we obtain

$$\begin{aligned} h + 0 + 0 + 0 - 0 - H_L &= 0 + \frac{v_2^2}{2g} \\ \Rightarrow h - H_L &= \frac{v_2^2}{2g} \\ \Rightarrow v_2^2 &= 2g(h - H_L) \\ \Rightarrow v_2 &= \sqrt{2g(h - H_L)} = \sqrt{2 \times 9.81(3.8 - 0.5)} = 8.05 \text{ m/s} \end{aligned}$$

The flow rate is given by

$$Q_2 = A_2 v_2 = \frac{\pi}{4} (30 \times 10^{-3})^2 \times 8.05$$

$$\Rightarrow Q_2 = 5.7 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\Rightarrow Q_2 = 5.7 \times 10^{-3} \times 10^3 \times 60$$

$$\Rightarrow Q_2 = 342 \text{ LPM}$$

Example 1.25

A siphon is made of a pipe whose inside diameter is 25.4 mm and is used to maintain a constant level in a 6.0975 m deep tank (Fig. 1.33). If the siphon discharge is 9.144 m below the top of tank, what will be the flow rate if the fluid level is 1.524 m below the top of tank?

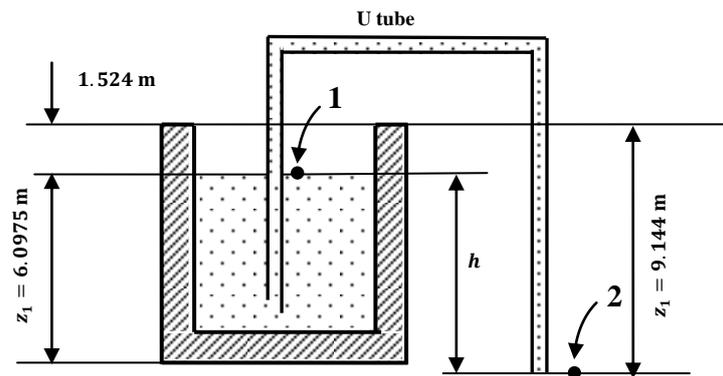


Figure 1.33

Solution: From Fig.1.33, $h = (9.144 - 1.524) = 7.62$ m. From the previous problem, we can write using the modified Bernoulli's theorem

$$v_2^2 = 2gh$$

$$\Rightarrow v_2 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 7.62} = 12.2 \text{ m/s}$$

Now

$$Q = v_2 \times A_2 = 12.2 \times \left(\frac{\pi}{4} 0.0254^2 \right)$$

$$= 0.00618 \text{ m}^3/\text{s} = 6.18 \text{ LPS}$$

Objective-Type Questions

Fill in the Blanks

1. Pascal's law states that the pressure exerted on a _____ is transmitted undiminished in _____ and acts with equal force on equal areas and at _____ to the surface of the container.
2. The total energy includes potential energy due to elevation and pressure and also _____.
3. The _____ energy of a particle of a fluid depends on its elevation above any arbitrary plane.
4. Pressure energy is possessed by the fluid per unit _____ of fluid virtue of the pressure under which the fluid exists.
5. Torricelli's theorem states that the velocity of the water jet of liquid is _____ proportional to the _____ of the head of the liquid producing it.

State True or False

1. Continuity equation states that the weight flow rate is the same for all cross sections of a pipe.
2. Hydraulic power is equal to the product of pressure and volume flow rate.
3. A pump converts mechanical energy into hydraulic energy by increasing the fluid flow.
4. It is easy to achieve overload protection using hydraulic systems.
5. Hydraulic power in kW = $\frac{\text{Flow (LPM)} \times \text{Pressure (bar)}}{320}$.

Review Questions

1. Define hydraulic power. Derive an expression for hydraulic power if the flow is in LPS and pressure in kPa.
2. How will you explain Pascal's law with reference to working of a hydraulic cylinder?
3. State Bernoulli's theorem.
4. What is a continuity equation and what are its implications relative to fluid power?
5. What is the significance of each term in the energy equation?
6. Define pressure head, elevation head and kinetic head.
7. State Torricelli's theorem and mention its significance.
8. Explain how a siphon operates.
9. State Pascal's law.
10. Explain the meaning of Bernoulli's equation and how it affects the flow of a fluid in a hydraulic circuit.
11. Relative to power, there is an analogy among mechanical, electrical and hydraulic systems. Describe this analogy.
12. What is the significance of each term in the energy equation?
13. State the basic principle laws and equations of hydraulics.

Answers

Fill in the Blanks

1. Confined fluid, all directions, right angles
2. Kinetic energy due to velocity
3. Potential
4. Weight
5. Directly, square root

State True or False

1. True
2. True
3. False
4. True
5. False