**Lecture 21**

**FLOW AND FORCE ANALYSIS OF VALVES**

**Learning Objectives**

Upon completion of this chapter, the student should be able to:

* Understand the mathematical equation for flow through anoverlapped four-way spool valve.
* Understand the mathematical equation for flow through an underlappedfour-way spool valve.
* Understand the mathematical equation for flow through athree-way critical center valve.
* Understand the mathematical equation for flow through athree-way open center valve.
* Describe the working and construction of a flapper nozzle valve and performits mathematical analysis.
* Analyze poppet, single-stage relief and pressure-compensated valves.
* Carry out mathematical analysis of valves used inhydraulic systems.

**1.1 Introduction**

Most hydraulic servo mechanisms or other high-performance systems rely for their operation on the metering of fluid through a valve. This chapter deals with a linearized method of analysis for “four-way valves.” They are called so because they have four connections, one for the supply pressure and another for the exhaust, and two control ports through both of which fluid may be metered, from the supply to either the system or the exhaust.

Metering valves are never fully open and their use is for accurately metering the flow of fluid through them. In this case of spool valves, longitudinal displacements of the spool are always small as compared with the spool’s diameter.

**1.2 Four-Way Spool Valves**

A spool valve used for metering purposes controls flow rate by throttling.Each port in a valve that is partially closed by a land on the spool becomes a control throttle.

The rate of flow of fluid through such a valve depends on the spool displacement from the null position “*x*” and on the pressure upstream and downstream of the valve. One way of representing the flow rate *q* through a valve is

 (1.1)

where*q*is the volume flow rate of the oil, *K*q is the flow gain, *K*c is the pressureflow coefficient and *p*mis the pressure difference across the load.Equation(1.1) implies that the flow rate is directly proportional to the valve opening and directly proportional to the pressure drop.

**1.2.1 Critical Center Valve**

In critical center valves, the lands of the spool are exactly of the same width as the annual ports of the valve body in the central or null position where the lands exactly cover the ports (Fig. 1.1).

***x***

**c**

***x***

**Load**

***x***

**Figure 1.1** Four-way valve**.**

***1.2.1.1 Flow Rate Prediction***

Orifices are a basic means for the control of fluid power. Flow characteristics of orifices play a major role in the design of many hydraulic control devices. An orifice is a sudden restriction of short length in a flow passage and may have a fixed or variable area. Two types of flow regime exist depending on whether inertial or viscous forces dominate. The flow through orifice must increase above that in the upstream region to satisfy the law of continuity. At high Reynolds number, the pressure drop across the orifice is caused by the acceleration of the fluid particles from the upstream velocity to the higher jet velocity. At low Reynolds numbers, the pressure drop is caused by the internal shear forces resulting from fluid viscosity.

The pressure difference required to accelerate the fluid particles from the lower upstream velocity to higher upstream velocity is found by applying Bernoulli’s equation and continuity equation. Here we will not derive the basic equations. Students are advised to go through any standard fluid mechanics books for the derivation of flow through orifice.

From fluid mechanics, flow through the orifice is given by

 (1.2)

(1.3)

Let us make following assumptions:

1. 
2. 
3. the supply pressure () is constant
4.  is negligible

By introducing the term, and using assumptions 2 and 4 we can write







Solving  and simultaneously we get

 (1.4)

 (1.5)

Using all assumptions in Eqs. (1.2) and (1.3) we get

 (1.6)

With the type of configuration illustrated in Fig. 1.1, it is usually accepted that

5/8 (1.7)

*ρ* = 870 kg/ (1.8)

Using values given in Eqs. (1.7) and (1.8) in Eq. (1.6) we get

 (1.9)

The assumption that the valve opening can be treated as orifices pre-supposes that  is small compared to so that the pressure drop across each orifice will be significant compared with . In practice the pressure difference across the load rarely exceeds 0.666. Allowing further simplification of Eq. (1.9) if, then less than 10% error is involved using the binomial approximation. Using binomial approximation, we have

 (1.10)

Hence Eq. (1.9) becomes

 (1.11)

andEq. (1.11) is similar in form to Eq. (1.1). Now

 (1.12)

 (1.13)

The above analysis predicts that the flow gain  can be treated as constant for a particular valve and supply pressure but the pressure flow coefficient  varies with the valve opening *x*. The variation of  is of minor significance for linear analysis.

In reality, spool lands never exactly match the annular ports in the valve body. Actual test results with a constant pressure drop across the valve ports show variations, particularly near the central or null position of the spool as those illustrated in Fig. 1.2.

The flow gain  is the slope of the approximate line in the figure, which can double its valve near null with negative lap. The magnitude of is the most important parameter of a valve and often also of any system incorporating the valve.

**Zero lap**

**Overlap**

**Underlap**

**Figure 1.2**Flow rates versus valve displacement for constant pressure drop.

**1.2.2 Open Center Valve (Underlapped Four-Way Valve)**

A valve in which the land of the spool never completely covers the ports of the valve body is said to be underlapped(or to have negative lap)(Fig. 1.3).

**(*u+x*)**

**(*u+x*)**

**d**

**c**

**b**

**a**

**(*u−x*)**

**Load**

**(*u−x*)**

**Load motion**

***x***

**Figure 1.3** Open center–four-way valve.

Referring to Fig. 1.3,a displacement of *x*(say to the left) unbalances the symmetry of the ports. Two of the annular orifices increase in width from *u* to *u + x* and two decrease from *u* to *u*−*x*.The flow ratesare estimated as follows:

(1.14)

 (1.15)

Let us make following assumptions:

1. 
2. 
3. The supply pressure () is constant
4.  is negligible

Writing, we can use these assumptions in Eqs. (1.14) and (1.15) and get

 (1.16)

that may be approximated as

 (1.17)

 (1.18)

Equation (1.18) is similar in form to Eq. (1.1):

 (1.19)

 (1.20)

The values refer to operation within the underlap region. Outside this region, these valves act as critical center valves with only two active ports. Note particularly that the flow gain is double that for a comparable critical center valve (Fig. 152) in the underlap region. Note also significant leakage flow when the valve is centered (leakage flow at null when the load flow  is zero)becomes .

**1.3 Three-Way Spool Valves**

Three-way valves have only one critical length dimension which helps to ease manufacture. However they cannot be used for hydraulic motors requiring flow reversal and are usually used in differential ram and is discussed below.

**1.3.1 Critical Center Valve**

The central spool position just closes both the supply and the exhaust port of the valve. A displacement of the spool to the left causes fluid to be metered into the ram chamber from the supply, whereas one to the right causes metering of fluid from the chamber to the exhaust (Fig. 1.4).

**c**

***x***

***x***

**Positive valve displacement**

**Positive load displacement**

**Figure 1.4** Three-way valve.

Oil flow rate is

*q*c = *C*d*π d*1*x*for*x*positive (1.21)

*q*c = *C*d*π d*1*x*for *x* negative (1.22)

Substituting *p*m­′ = *p*c −*p*s/2 we get

*q*c = *C*d*π d*1*x*for *x* positive (1.23)

*q*c = *C*d*π d*1*x*for *x* negative (1.24)

thatmay be approximated.Noting that

= ≈  (1.25)

we get

*q*c = *C*d*π d*1*x**C*d*π d*1*x* (1.25)

where the positive sign is associated with negative values of x and negative sign with positive ones.Using *C*d6.7 for SI units with pressure in bars we get

*K*q= 6.7 *π d*1 (1.26)

*K*c= 6.7 *π d*1(1.27)

**1.3.2 Open Center Valve (Underlapped Three-Way Valve)**

***x***

**(*u−x*)**

**(*u+x*)**

**Figure 1.5** Open center–three-way valve.

Referring to Fig. 1.5, The orifice equation for *q*1 can be written as

*q*1 = *C*d*π d*1(*u* + *x*) (1.28)

Using binomial approximation we can write

*q*2 ≈*C*d*π d*1(*u* + *x*) (1.29)

Similarly orifice equation for *q*2 can be written as

*q*2 = *C*d*π d*1(*u* − *x*) (1.30)

Using binomial approximation we can write

*q*2 ≈ *C*d*π d*1(*u* − *x*) (1.31)

The flow rate*q*c is the difference of *q*1 and *q*2, that is

*q*c = *q*1 – *q*2

≈ *C*d*π d*12*x* − *C*d*π d*1 (1.32)

Comparing with Eq. (1.1) we can write

*K*q= 13.4 *π d*1(1.33)

*K*q= 13.4 *π d*1*u* (1.34)

**1.4 Flapper Nozzle Valve**

Commonly used asthe first stage of two-stage servo valves, nozzles and the fixed upstream orifices used with them are made with diameters and  in the range 0.2–0.8 mm and the distance between each nozzle and the flapper in a double valve is often less than 0.2 mm.Each nozzle and orifice is as nearly a sharp-edged orifice as possible and treated as such for analytical purposes. The curtain area formed by the flapper at a nozzle exit modulates the control pressure caused by the fixed upstream orifice.

**Load**

**Orifice area diameter**

**Flapper**

**Figure 1.6** Flapper nozzle valve.

Consider a flopper nozzle valve indicated in Fig. 1.6. We assume that the valve has a balanced condition such that *x* = 0 and = 0, *q* = 0. This occurs with the pressure downstream of each of the fixed orifices iequal to*p*s/2 and when the flow through each orifice equals that through each nozzle, that is

 (steady state) = (steady state) = (1.35)

(steady state) = (steady state) = (1.36)

and 

This also implies that the orifice size and the curtain area in the null position are approximately equal

=

Let us assume feasible ratio of

= (1.37)

Consideringthat the valve is not in balance, that is,*x* has the same value as  has, we have

*q* =

 (1.38)

where

=

Also

*q* =

(1.39)

Using binomial approximations, we have

 (1.40)

 (1.41)

Adding and dividing by 2, we get

*q*= + (1.42)

thatis in the form of. Now

=

=

=

**1.5 Special-Purpose Valves**

The dynamic characteristics of hydraulic devices such as relief valves or flow-control valves are not well understood. For design purposes, it is useful to know the likely influence of spool mass, spring rate, orifice size or other parameter on the response of any particular valve to changes in pressure or flow rate or to other disturbances. A pressure-control valve and a flow-control valve are considered below.

**1.5.1 Poppet Valves**

Pressure control valve and flow control valves employs a poppet valve (Fig. 1.7) and their characteristics will be influenced by the flow pattern existing at or near the valve seat.

**Figure 1.7** Poppet valve.

For a poppet displacement *x*, the area of flow is *C*d*x π d*, where *C*d is the flow coefficient. For a pressure drop of across the orifice formed between the seat and poppet, the fluid velocity *v* is taken as equal to

 (1.43)

and the volume flow rate as

*q*= *C*d*x π d* (1.44)

The momentum of the jet has an axial component equal to *ṁv*(where *ṁ* is the mass flow rate) that may be written as

Axial component of momentum of jet ( = Mass flow rate ×Velocity

= Density ×Volume flow rate ×Velocity component

 (1.45)

Please checkEq. (1.45)

where

 = *C*d*π d* (1.46)

The axial component of jet momentum is*x.*

**1.5.2 Single-Stage Relief Valve**

Spring loaded valve is illustrated in Fig. 1.8. Let the pressure applied to valve to open is *p*It is the effects of small changes in this pressure from  to that are to be considered. The outlet (exhaust) pressure is assumed to be zero throughout.

**(*m*)**

**()**

**Figure 1.8** Relief valve.

With the poppet closed (i.e.,*x* = 0), there is some spring-related preload force holding it down that is designated as *F*. Under a steady-state condition with a greater pressure applied than that needed to overcome the preload, the valve partly opened and the poppet stationary, there is a steady-state balance of forces relating this pressure to the poppet displacement. For the applied pressure *p*0 and the poppet displacement *x*0, the relation is

= *F*+ *k*s*x*0+*x*0 (1.47)

If the approach velocity is negligible then , given in Eq. (1.46), and the fluid frictional drag force across the poppet face is neglected.

For some other steady-state position of the poppet (displacement), another pressure would occur according to

() *F*+*k*s() + ()() (1.48)

Note that Eqs. (1.47) and (1.48) refer to steady state with poppet stationary. Under dynamic conditions with the poppet moving, the balance of forces has to take into account the effective mass of the poppet *m* and any damping (assumed viscous of rate *f*) as well as spring, pressure and momentum forces. For a pressure and a poppet displacement, the balance forces become

() *F*+ *k*s*x*0 +*x*0 +*x*0 + (*k*s+*p*0) *x* + *f Dx* + *mD*2*x* (1.49)

Assume that the terms involvingare negligibly small. Subtracting Eq. (1.47) from Eq. (1.49), we get

−*x*0= (*k*s+) *x* +*f Dx* + *mD*2*x*

Indicating a second-order relation between changes in pressure  and changes in poppet displacement *x* given by

 (1.50)

where



= (*k*s+) /*m*

This relation suggests that any oscillation of the poppet is associated with much stiffer spring than the physical spring constant would suggest. As a numerical example, consider a valve of diameter *d* = 6mm for use at a nominal pressure of = 70 bar. The projected area equals 28 , so that the value of *F* is approximately 20 N andif this is obtained by initially compressing the spring 10mm, the spring rate would have a valve of *k*s = 20 N/mm. Assume that the poppet has a 90° cone angle and flow coefficient  = 0.7, then  for the valve is about 13.2 mm and  = 92.4 N/mm. The effective spring rate is not 20 but 112.4 N/mm. If the effective mass were 0.001 kg, then the natural frequency of poppet oscillations would be 533 Hz.

**1.6 Pressure-Compensated Flow-Control Valve**

Figure 1.9 illustrates a pressure-compensated flow-control valve which is designed to pass a constant flow rate of fluid despite fluctuations of the inlet and outlet pressures. The device has two orifices in series:one is preset manually to select the desired flow rate, while the other varies with the pressure difference across the valve. The aim is to keep the flow rate constant by maintaining a constant pressure difference across the present orifice.

For analysis, changes in the outlet pressure  represent the disturbances externally imposed on the device with the inlet (supply) pressure assumed constant. This simulates a meter-in control with the poppet valve partly open and this analysis concerns small changes in this valve opening. The fully open or fully closed conditions are not dealt with. The datum for spring force acting on the spool is taken from some arbitrary position of the spool represented by the poppet (i.e., control orifice) opening of 

**Manually preset orifice**

**Spring**

**Oil inlet**

**Annular control orifice**

**Oil outlet**

**Moving spool**

**Figure 1.9** Pressure-compensated flow-control valve.

**1.6.1 Forces**

The steady-state balance of forces for some equilibrium operating position with the poppet stationary and open distance, for a supply pressure , outlet pressure  and consequent chamber pressure  is given by

(−)= *F*−(−) (1.51)

Under dynamic conditions with the spool in motion for the outlet pressure + , with the instantaneous spool position distance *x* to the left of its initial position, noting that +  is the instantaneous chamber pressure, the balance of forces is given by

{(+ ) −(+ } = *F*−*x*– { –(+)}(*x*+ )−*f Dx* – *mD*2*x* (1.52)

Assuming that and *x*are small and the terms may be neglected, subtracting Eq. (1.52) from Eq. (1.51) gives

−= *x* + *Dx* + *D*2*x*− (1.53)

**1.6.2 Flow Rates**

Under steady-state condition, the flow rate through control orifice must be equal to that through the preset orifice. Hence, for the steady conditions previously used with the constant , the outlet , the chamber pressure  and control orifice opening  using the subscript e for the preset orifice,

*q*0 = *C*d*x*0*π d* = *C*de*a*e (1.54)

Under dynamic conditions, the flow rate through the control orifice will no longer be equal – the flow through the control orifice isand that through preset orifice . The difference between the two flow rates (into and out from the valve chamber) causes compression of the oil in the valve chamber or

 (1.55)

where*V* is the volume of oil and is the effective bulk modules. In Eq. (1.55),*V* is assumed to be constant.The flow rate through the control orifice may be written as

 (1.56)

Subtracting Eq.(1.54)from Eq. (1.56) neglecting the terms involving  we get

= −+ *x* +*A*p*Dx* (1.57)

and similarly for the change in flow rate through the preset orifice,

= =− (1.58)

Now subtracting Eq. (1.58) from Eq.(1.57) gives

=*Dx*+*x*+  (1.59)

Also equating Eqs. (1.55) and (1.59) after substituting Eq. (1.53) for and after differentiating, for *D*to obtain and*D*terms of and *x* leads to (closed loop) relation between displacements of the spool and changes of the outlet pressure

*x* + x + *x* + *x* = *K*+ *TD*(1.60)

 (1.61)

where











= 

= 



This linear equation is obviously difficult to apply particularly as the coefficients depend on the initial conditions that might be classed as the “normal” operating conditions. In practice,the valves of this type can be unsatisfactory at initial stage. The relation does indicate that a value is stable if  is greater than and confirms that increasing the viscous damping helps to stabilize the valve, increasing both and .

**Example 1.1**

A four-way valve with full periphery annular ports has a 6 mm diameter spool and it may be assumed that the spool lands fully cover the valve ports in zero or mid-position. Estimate the flow rate through one port when the pressure drop across it is 70 bar for every mm of spool displacement.

**Solution**

Spool diameter (*d*) = 6mm

Pressure drop (Δ*p*) = 70 bar

Spool displacement (*x*) = 1mm

By the orifice flow equation, theflow rate is



Letting *C*d = 5/8 and *ρ* = 870 kg/m3 and Δ*p*in bar, we have



Therefore,

*q*= 6.7*π*(6 × 10−3) × (1 × 10−3) × 

*q*= 1.494 × 10−3 m3/s

**Example1.2**

What would be the flow coefficient *K*q of the above valve if it is used as part of a servo system having oil supply pressure: (a) 140 bar, (b) 210 bar?

**Solution:**

(a) Supply pressure (*p*s) = 140 bar. We know . Therefore,

*K*q = 6.7×*π*(6 × 10−3) ×  = 1.494 m3/s/mm

(b)Supply pressure (*p*s) = 210 bar. We know . Therefore,

*K*q = 6.7 × *π* (6 × 10−3) × = 1.83 m3/s/mm

**Example1.3**

A three-way spool valve with half the annular periphery of the valve port blocked off and spool diameter 9 mm is used in a system supplied with oil at 120 bar pressure. The half-area piston has areas 0.004 m2 and 0.002 m2 and a maximum required velocity 0.3 m/s. Estimate the maximum spool displacement required. (Assume one-third pressure drop through valve.)

**Solution:** Supply pressure (*p*s) = 120 bar. Spool diameter (*d*) = 9mm. Piston area (*A*p) = 0.004m2. Maximum velocity (*v*) = 0.3m/s. Maximum valve flow rate is

*Q* = *A*p× *v* = 0.004×0.3 = 1.2 × 10–3 m3/s

Pressure difference across load (*p*m) = 40 bar (one-third of supply pressure).

For a three-way valve with half-area piston, the pressure drop is

Δ*p* = (*p*s/2) −*p*m = 20 bar

Also from the orifice equation we can write

*q*= 6.7 × *π* (9 × 10–3) × *x* × 

*q*= 1.198 × *x*

Equating two values of *Q and q*, we have

1.2 × 10−3 = 1.198 × *x*

*x* = 1.0016mm.

As half the annular periphery of valve is blocked, for the same flow rate, the displacement required is twice the calculated value.Therefore, the valve displacement is

*y* = 2 × *x* = 2.003mm

**Example 1.4**

In a 240-bar servo system employing a four-way valve, a valve underlap is used to assist in damping system oscillations. The valve has a 4-mm-diameter spool, full periphery ports and nominal underlap of 0.0127 mm. Estimate the pressureflow coefficient *K*c for the valve.

**Solution:** Supply pressure (*p*s) = 240 bar. Spool diameter (*d*) = 4mm. Underlap(*u*) = 0.0127mm. We know



**Objective-Type Questions**

**Fill in the Blanks**

1. In critical center valves, the lands of the spool are of the \_\_\_\_\_\_ as the annual ports of the valve body in the central or null position where the lands exactly cover the ports.

2. A valve in which the land of the spool never completely covers the ports of the valve body is said to be \_\_\_\_\_\_.

3. \_\_\_\_\_\_ is commonly used as the first stage of two-stage servo valves.

4. A pressure-compensated flow control valve is designed to pass a \_\_\_\_\_\_ of fluid despite fluctuations of the inlet and outlet pressures.

5. Flow gain can be treated as \_\_\_\_\_\_ for a particular valve and supply pressure.

**State True or False**

1. The central spool position just closes both the supply and the exhaust port of the valve.

2. Most hydraulic servo mechanisms or other high-performance systems rely for their operation on the metering of fluid through a valve.

3. Metering valves are always fully open and their use is for accurately metering the flow of fluid through them.

4. A spool valve used for metering purposes controls flow rate by throttling.

5. A pressure flow coefficient will vary with valve opening and the variation of pressure flow coefficient are of major significance for linear analysis.

**Review Questions**

**1**. Discuss in detail the flow–displacement relationship of a critical center spool valve, giving its possible applications in hydraulic control valves.

**2**. Discuss in detail the flow forces acting on apressure-compensated flow control valve and derive an expression for these forces.

**3**. Derive an expression for the flow–displacement relations of anunderlapped four-way valve.

**4**. Derive an expression for the flow for a flapper nozzle valve.

**5**.What is the major limitation of a flapper nozzle amplifier and how it can be overcome?

**6.**Why is a feedback system used in a multistage servo valve?

7. What is a critical center valve?

8. What does an underlapped four-way valve mean?

9. Give two applications of a flapper nozzle valve.

10. How is a higher flow rate achieved in an electrohydraulic servo valve?

11. Why is a feedback system used in a multistage servo valve?

12. Illustrate the two different types of feedbacks used in a multistage electrohydraulic valve?

13. What is the major limitation of a flapper nozzle amplifier and how it can be overcome?

**Answers**

**Fill in the Blanks**

1.Same width

2.Underlapped

3.Flapper nozzle valve

4.Constant flow rate

5.Constant

**State True or False**

1.True

2.True

3.False

4.True

5.False